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0.1 Crystal Structures

$$\mathbf{R} = m\mathbf{a}_1 + n\mathbf{a}_2, \quad (1)$$

$$\mathbf{R} = m\mathbf{a}_1 + n\mathbf{a}_2 + o\mathbf{a}_3. \quad (2)$$

$$n\lambda = 2d \sin \theta. \quad (3)$$

$$\mathcal{E}(\mathbf{r}, t) = \mathcal{E}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}. \quad (4)$$

$$I(\mathbf{r}) = |\mathcal{E}_0 e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}|^2 = |\mathcal{E}_0|^2. \quad (5)$$

$$\mathcal{E}(\mathbf{r}, t) = \mathcal{E}_0 e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{R}) - i\omega t}. \quad (6)$$

$$\mathcal{E}(\mathbf{r}, t) \propto e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{R})} e^{-i\omega t}. \quad (7)$$

$$\mathcal{E}(\mathbf{R}', t) \propto \mathcal{E}(\mathbf{r}, t) \rho(\mathbf{r}) e^{i\mathbf{k}'\cdot(\mathbf{R}'-\mathbf{r})}. \quad (8)$$

$$\mathcal{E}(\mathbf{R}', t) \propto e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{R})} \rho(\mathbf{r}) e^{i\mathbf{k}'\cdot(\mathbf{R}'-\mathbf{r})} e^{-i\omega t} = e^{i(\mathbf{k}'\cdot\mathbf{R}' - \mathbf{k}\cdot\mathbf{R})} \rho(\mathbf{r}) e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}} e^{-i\omega t}. \quad (9)$$

$$\mathcal{E}(\mathbf{R}', t) \propto e^{-i\omega t} \int_V \rho(\mathbf{r}) e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}} dV. \quad (10)$$

$$I(\mathbf{K}) \propto |e^{-i\omega t} \int_V \rho(\mathbf{r}) e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}} dV|^2 = \left| \int_V \rho(\mathbf{r}) e^{-i\mathbf{K}\cdot\mathbf{r}} dV \right|^2, \quad (11)$$

$$\mathbf{R} = m\mathbf{a}_1 + n\mathbf{a}_2 + o\mathbf{a}_3, \quad (12)$$

$$\mathbf{R} \cdot \mathbf{G} = 2\pi l, \quad (13)$$

$$e^{i\mathbf{G}\cdot\mathbf{R}} = 1. \quad (14)$$

$$\mathbf{G} = m'\mathbf{b}_1 + n'\mathbf{b}_2 + o'\mathbf{b}_3, \quad (15)$$

$$\mathbf{b}_1 = 2\pi \frac{\mathbf{a}_2 \times \mathbf{a}_3}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)}, \quad \mathbf{b}_2 = 2\pi \frac{\mathbf{a}_3 \times \mathbf{a}_1}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)}, \quad \mathbf{b}_3 = 2\pi \frac{\mathbf{a}_1 \times \mathbf{a}_2}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)}. \quad (16)$$

$$\mathbf{a}_i \cdot \mathbf{b}_j = 2\pi \delta_{ij}, \quad (17)$$

$$e^{i\mathbf{G} \cdot \mathbf{r}} = e^{i\mathbf{G} \cdot \mathbf{r}} e^{i\mathbf{G} \cdot \mathbf{R}} = e^{i\mathbf{G} \cdot (\mathbf{r} + \mathbf{R})}. \quad (18)$$

$$\rho(x) = C + \sum_{n=1}^{\infty} \left\{ C_n \cos(x2\pi n/a) + S_n \sin(x2\pi n/a) \right\} \quad (19)$$

$$\rho(x) = \sum_{n=-\infty}^{\infty} \rho_n e^{inx2\pi/a}, \quad (20)$$

$$\rho_{-n}^* = \rho_n, \quad (21)$$

$$g = n \frac{2\pi}{a}, \quad (22)$$

$$\rho(\mathbf{r}) = \sum_{\mathbf{G}} \rho_{\mathbf{G}} e^{i\mathbf{G} \cdot \mathbf{r}}, \quad (23)$$

$$I(\mathbf{K}) \propto \left| \sum_{\mathbf{G}} \rho_{\mathbf{G}} \int_V e^{i(\mathbf{G} - \mathbf{K}) \cdot \mathbf{r}} dV \right|^2. \quad (24)$$

$$\mathbf{K} = \mathbf{k}' - \mathbf{k} = \mathbf{G}, \quad (25)$$

$$I(\mathbf{G}) \propto \left| \int_V \rho(\mathbf{r}) e^{-i\mathbf{G} \cdot \mathbf{r}} dV \right|^2. \quad (26)$$

$$I(\mathbf{G}) \propto \left| \sum_{\mathbf{R}} \int_{V_{cell}} \rho(\mathbf{r} + \mathbf{R}) e^{-i\mathbf{G} \cdot (\mathbf{r} + \mathbf{R})} dV \right|^2 = |N \int_{V_{cell}} \rho(\mathbf{r}) e^{-i\mathbf{G} \cdot \mathbf{r}} dV|^2, \quad (27)$$

$$\rho(\mathbf{r}) = \sum_i \rho_i(\mathbf{r} - \mathbf{r}_i), \quad (28)$$

$$\int_{V_{cell}} \rho(\mathbf{r}) e^{-i\mathbf{G} \cdot \mathbf{r}} dV = \sum_i e^{-i\mathbf{G} \cdot \mathbf{r}_i} \int_{V_{atom}} \rho_i(\mathbf{r}') e^{-i\mathbf{G} \cdot \mathbf{r}'} dV', \quad (29)$$

$$k'_{\perp} - k_{\perp} = 2k_{\perp} = 2 \frac{2\pi}{\lambda} \sin \Theta = G_{\perp}, \quad (30)$$

0.2 Bonding in Solids

$$\phi(r) = \frac{A}{r^n} - \frac{B}{r^m}, \quad (31)$$

$$E_{Na} = -1.748 \frac{e^2}{4\pi\epsilon_0 a} = -M_d \frac{e^2}{4\pi\epsilon_0 a}. \quad (32)$$

$$H = -\frac{\hbar^2 \nabla_1^2}{2m_e} - \frac{\hbar^2 \nabla_2^2}{2m_e} + \frac{e^2}{4\pi\epsilon_0} \left\{ \frac{1}{R} + \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|} - \frac{1}{|\mathbf{R}_A - \mathbf{r}_1|} - \frac{1}{|\mathbf{R}_B - \mathbf{r}_2|} - \frac{1}{|\mathbf{R}_A - \mathbf{r}_2|} - \frac{1}{|\mathbf{R}_B - \mathbf{r}_1|} \right\}, \quad (33)$$

$$\Psi_{\uparrow\downarrow}(\mathbf{r}_1, \mathbf{r}_2) \propto \phi_A(\mathbf{r}_1)\phi_B(\mathbf{r}_2) + \phi_A(\mathbf{r}_2)\phi_B(\mathbf{r}_1) \quad (34)$$

$$\Psi_{\uparrow\uparrow}(\mathbf{r}_1, \mathbf{r}_2) \propto \phi_A(\mathbf{r}_1)\phi_B(\mathbf{r}_2) - \phi_A(\mathbf{r}_2)\phi_B(\mathbf{r}_1), \quad (35)$$

$$E = \frac{\int \Psi^*(\mathbf{r}_1, \mathbf{r}_2)H\Psi(\mathbf{r}_1, \mathbf{r}_2)d\mathbf{r}_1d\mathbf{r}_2}{\int \Psi^*(\mathbf{r}_1, \mathbf{r}_2)\Psi(\mathbf{r}_1, \mathbf{r}_2)d\mathbf{r}_1d\mathbf{r}_2}. \quad (36)$$

$$E_{\text{singlet}} = 2E_0 + \Delta E_{\uparrow\downarrow}, \quad (37)$$

$$E_{\text{triplet}} = 2E_0 + \Delta E_{\uparrow\uparrow}. \quad (38)$$

$$E = 2E_0 + C \pm X, \quad (39)$$

0.3 Mechanical Properties

$$Y = \frac{\sigma}{\epsilon} = \frac{F}{A} \frac{l}{\Delta l}. \quad (40)$$

$$\sigma = Y\epsilon. \quad (41)$$

$$F = \frac{YA}{l}\Delta l, \quad (42)$$

$$G = \frac{\tau}{\alpha}. \quad (43)$$

$$K = -p \frac{V}{\Delta V}, \quad (44)$$

$$\frac{\Delta l_2}{l_2} = \frac{\Delta l_3}{l_3} = -\nu \frac{\Delta l_1}{l_1} = -\nu\epsilon. \quad (45)$$

$$(l_1 + \Delta l_1)(l_2 + \Delta l_2)(l_3 + \Delta l_3). \quad (46)$$

$$l_1l_2l_3 + \Delta l_1l_2l_3 + l_1\Delta l_2l_3 + l_1l_2\Delta l_3. \quad (47)$$

$$\begin{aligned} \Delta l_1l_2l_3 + l_1\Delta l_2l_3 + l_1l_2\Delta l_3 &= \Delta l_1l_2l_3 + l_1(-\nu \frac{\Delta l_1}{l_1}l_2)l_3 + l_1l_2(-\nu \frac{\Delta l_1}{l_1}l_3) \\ &= (1 - 2\nu)\Delta l_1l_2l_3. \end{aligned} \quad (48)$$

$$G = \frac{Y}{2(1 + \nu)}. \quad (49)$$

$$\phi(x) = \phi(a) + \frac{\phi'(a)}{1!}(x-a) + \frac{\phi''(a)}{2!}(x-a)^2 + \frac{\phi'''(a)}{3!}(x-a)^3 + \dots \quad (50)$$

$$\alpha = \tan^{-1}\left(\frac{x}{a}\right) \approx \frac{x}{a}. \quad (51)$$

$$\tau = G\alpha \approx \frac{Gx}{a}, \quad (52)$$

$$\tau = C \sin\left(\frac{2\pi x}{b}\right), \quad (53)$$

$$C \frac{2\pi x}{b} = \frac{Gx}{a}, \quad (54)$$

$$C = \tau_Y = \frac{Gb}{2\pi a}. \quad (55)$$

0.4 Thermal Properties of the Lattice

$$M \frac{d^2 x}{dt^2} = -\gamma x, \quad (56)$$

$$\omega = \sqrt{\frac{\gamma}{M}}. \quad (57)$$

$$E = \frac{1}{2} M v^2 + \frac{1}{2} \gamma x^2. \quad (58)$$

$$\frac{1}{2} \gamma x_{\max}^2 = k_B T, \quad (59)$$

$$x_{\max} = \left(\frac{2k_B T}{\gamma} \right)^{1/2}. \quad (60)$$

$$M \frac{d^2 u_n}{dt^2} = -\gamma(u_n - u_{n-1}) + \gamma(u_{n+1} - u_n), \quad (61)$$

$$M \frac{d^2 u_n}{dt^2} = -\gamma[2u_n - u_{n-1} - u_{n+1}]. \quad (62)$$

$$u_n(t) = u e^{i(kan - \omega t)}, \quad (63)$$

$$-M\omega^2 e^{i(kan - \omega t)} = -\gamma[2 - e^{-ika} - e^{ika}]e^{i(kan - \omega t)} = -2\gamma(1 - \cos ka)e^{i(kan - \omega t)}, \quad (64)$$

$$\omega(k) = \sqrt{\frac{2\gamma(1 - \cos ka)}{M}} = 2\sqrt{\frac{\gamma}{M}} \left| \sin \frac{ka}{2} \right|. \quad (65)$$

$$\omega(k) = \sqrt{\frac{\gamma}{M}} ak = \nu k, \quad (66)$$

$$M_1 \frac{d^2 u_n}{dt^2} = -\gamma[2u_n - v_{n-1} - v_n], \quad M_2 \frac{d^2 v_n}{dt^2} = -\gamma[2v_n - u_n - u_{n+1}], \quad (67)$$

$$u_n(t) = u e^{i(kbn - \omega t)}, \quad v_n(t) = v e^{i(kbn - \omega t)}. \quad (68)$$

$$-\omega^2 M_1 u = \gamma v(1 + e^{-ikb}) - 2\gamma u, \quad -\omega^2 M_2 v = \gamma u(e^{ikb} + 1) - 2\gamma v. \quad (69)$$

$$\begin{vmatrix} 2\gamma - \omega^2 M_1 & -\gamma(e^{-ikb} + 1) \\ -\gamma(1 + e^{ikb}) & 2\gamma - \omega^2 M_2 \end{vmatrix} = 0. \quad (70)$$

$$\omega^2 = \gamma \left(\frac{1}{M_1} + \frac{1}{M_2} \right) \pm \gamma \left[\left(\frac{1}{M_1} + \frac{1}{M_2} \right)^2 - \frac{4}{M_1 M_2} \sin^2 \frac{kb}{2} \right]^{1/2}, \quad (71)$$

$$u_{N+n}(t) = u_n(t). \quad (72)$$

$$e^{ikan} = e^{ika(N+n)}, \quad (73)$$

$$e^{ikNa} = 1, \quad (74)$$

$$k = \frac{2\pi}{aN} m, \quad (75)$$

$$E_l = \left(l + \frac{1}{2} \right) \hbar \omega, \quad (76)$$

$$E_l(k) = \left(l + \frac{1}{2} \right) \hbar \omega(k). \quad (77)$$

$$\mathbf{k} = (k_x, k_y, k_z) = \frac{2\pi}{aN} (n_x, n_y, n_z) = \left(\frac{n_x 2\pi}{L}, \frac{n_y 2\pi}{L}, \frac{n_z 2\pi}{L} \right), \quad (78)$$

$$F = \gamma \Delta a, \quad (79)$$

$$\sigma = \frac{\gamma \Delta a}{a^2}. \quad (80)$$

$$Y = \frac{\sigma}{\epsilon} = \frac{\gamma \Delta a}{a^2} \frac{a}{\Delta a} = \frac{\gamma}{a}. \quad (81)$$

$$E_n = \left(n + \frac{1}{2} \right) \hbar \omega_E. \quad (82)$$

$$\langle E \rangle = 3N_A \left(\langle n \rangle + \frac{1}{2} \right) \hbar \omega_E. \quad (83)$$

$$\langle n \rangle = \frac{1}{e^{\hbar \omega_E / k_B T} - 1}. \quad (84)$$

$$\langle E \rangle = 3N_A \left(\frac{1}{e^{\hbar \omega_E / k_B T} - 1} + \frac{1}{2} \right) \hbar \omega_E. \quad (85)$$

$$C = \frac{\partial \langle E \rangle}{\partial T} = 3R \left(\frac{\hbar \omega_E}{k_B T} \right)^2 \frac{e^{\hbar \omega_E / k_B T}}{(e^{\hbar \omega_E / k_B T} - 1)^2}. \quad (86)$$

$$p_1 \propto e^{-\hbar \omega_E / k_B T}, \quad (87)$$

$$\langle E \rangle = \frac{\hbar \omega}{e^{\hbar \omega / k_B T} - 1} \quad (88)$$

$$\langle E \rangle = 3 \int_0^{\omega_D} \frac{\hbar \omega}{e^{\hbar \omega / k_B T} - 1} d\omega, \quad (89)$$

$$\langle E \rangle = 3 \int_0^{\omega_D} \frac{g(\omega) \hbar \omega}{e^{\hbar \omega / k_B T} - 1} d\omega \quad (90)$$

$$N = \frac{4}{3} \pi n_{\max}^3, \quad (91)$$

$$N = \frac{4}{3} \pi \left(\frac{L k_{\max}}{2\pi} \right)^3. \quad (92)$$

$$N(\omega) = \frac{4}{3} \pi \left(\frac{L\omega}{2\pi\nu} \right)^3 = \frac{V}{6\pi^2\nu^3} \omega^3, \quad (93)$$

$$g(\omega) = \frac{dN}{d\omega} = \frac{\omega^2 V}{2\pi^2\nu^3}. \quad (94)$$

$$3N = 3 \int_0^{\omega_D} g(\omega) d\omega. \quad (95)$$

$$\omega_D^3 = 6\pi^2 \frac{N}{V} \nu^3. \quad (96)$$

$$\langle E \rangle = 3 \int_0^{\omega_D} \frac{\omega^2 V}{2\pi^2\nu^3} \frac{\hbar \omega}{e^{\hbar \omega / k_B T} - 1} d\omega = \frac{3V\hbar}{2\pi^2\nu^3} \int_0^{\omega_D} \frac{\omega^3}{e^{\hbar \omega / k_B T} - 1} d\omega, \quad (97)$$

$$\langle E \rangle = \frac{3V k_B^4 T^4}{2\pi^2\nu^3 \hbar^3} \int_0^{x_D} \frac{x^3}{e^x - 1} dx = 9N k_B T \left(\frac{T}{\Theta_D} \right)^3 \int_0^{x_D} \frac{x^3}{e^x - 1} dx. \quad (98)$$

$$C = \frac{12\pi^4}{5} Nk_B \left(\frac{T}{\Theta_D} \right)^3. \quad (99)$$

$$\kappa = \kappa_p + \kappa_e. \quad (100)$$

$$\kappa = \frac{1}{A} \frac{\partial Q}{\partial t} \frac{\Delta x}{\Delta T}. \quad (101)$$

$$\kappa_p = \frac{1}{3} c \lambda_p v_p, \quad (102)$$

$$\frac{\Delta l}{l} = \alpha \Delta T, \quad (103)$$

$$G = U + PV - TS. \quad (104)$$

$$T_m = \frac{(0.05a)^2 \gamma}{2k_B} = \frac{(0.05a)^2 \omega^2 M}{2k_B}. \quad (105)$$

$$T_m = \frac{(0.05a)^2 \Theta_D^2 k_B M}{2\hbar^2}. \quad (106)$$

0.5 Electronic Properties of Metals: Classical Approach

$$\frac{1}{2} m_e v_t^2 = \frac{3}{2} k_B T. \quad (107)$$

$$m_e \frac{d\mathbf{v}}{dt} = -e\mathcal{E}, \quad (108)$$

$$\mathbf{v}(t) = \frac{-e\mathcal{E}t}{m_e}, \quad (109)$$

$$\bar{\mathbf{v}} = \frac{-e\mathcal{E}\tau}{m_e}. \quad (110)$$

$$n|\bar{\mathbf{v}}|A. \quad (111)$$

$$-en|\bar{\mathbf{v}}|A. \quad (112)$$

$$\mathbf{j} = -en\bar{\mathbf{v}}, \quad (113)$$

$$\mathbf{j} = \frac{ne^2\tau}{m_e} \mathcal{E} = \sigma \mathcal{E} = \frac{\mathcal{E}}{\rho}, \quad (114)$$

$$\sigma = \frac{ne^2\tau}{m_e}, \quad (115)$$

$$\rho = \frac{m_e}{ne^2\tau}. \quad (116)$$

$$\mu = \frac{e\tau}{m_e}, \quad (117)$$

$$\sigma = n\mu e, \quad \rho = \frac{1}{n\mu e}. \quad (118)$$

$$\mathcal{E}_H = R_H j_x B_z, \quad (119)$$

$$|-e\mathcal{E}_H| = |-eB_z v_x|. \quad (120)$$

$$R_H = \frac{\mathcal{E}_H}{j_x B_z} = \frac{\mathcal{E}_H}{-env_x B_z} = \frac{v_x B_z}{-env_x B_z} = \frac{-1}{ne}. \quad (121)$$

$$R_H = \frac{1}{pe}, \quad (122)$$

$$\mathcal{E}(z, t) = \mathcal{E}_0 e^{i(kz - \omega t)}, \quad (123)$$

$$k = \frac{2\pi N}{\lambda_0}, \quad (124)$$

$$N = n + i\kappa \quad (125)$$

$$N = \sqrt{\epsilon} = \sqrt{\epsilon_r + i\epsilon_i}, \quad (126)$$

$$\mathcal{E}(z, t) = \mathcal{E}_0 e^{i((2\pi N/\lambda_0)z - \omega t)} = \mathcal{E}_0 e^{i((\omega\sqrt{\epsilon}/c)z - \omega t)}. \quad (127)$$

$$m_e \frac{d^2 x(t)}{dt^2} = -e\mathcal{E}_0 e^{-i\omega t}. \quad (128)$$

$$x(t) = Ae^{-i\omega t}, \quad (129)$$

$$A = \frac{e\mathcal{E}_0}{m_e \omega^2}. \quad (130)$$

$$P(t) = -nex(t) = -neAe^{-i\omega t} = -\frac{ne^2 \mathcal{E}_0 e^{-i\omega t}}{m_e \omega^2}. \quad (131)$$

$$D = \epsilon\epsilon_0 \mathcal{E} = \epsilon_0 \mathcal{E} + P, \quad (132)$$

$$\epsilon = 1 + \frac{P(t)}{\epsilon_0 \mathcal{E}_0 e^{-i\omega t}}. \quad (133)$$

$$\epsilon = 1 - \frac{ne^2}{\epsilon_0 m_e \omega^2} = 1 - \frac{\omega_P^2}{\omega^2}, \quad (134)$$

$$\omega_P^2 = \frac{ne^2}{m_e \epsilon_0}. \quad (135)$$

$$\frac{\kappa}{\sigma} = LT, \quad (136)$$

$$\frac{\kappa}{\sigma} = \frac{3}{2} \frac{k_B^2}{e^2} T = LT, \quad (137)$$

0.6 Electronic Properties of Solids: Quantum Mechanical Approach

$$-\frac{\hbar^2 \nabla^2}{2m_e} \psi(\mathbf{r}) + U(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r}). \quad (138)$$

$$U(\mathbf{r}) = U(\mathbf{r} + \mathbf{R}), \quad (139)$$

$$\psi(\mathbf{r}) = \psi(x, y, z) = \psi(x + L, y, z) = \psi(x, y + L, z) = \psi(x, y, z + L). \quad (140)$$

$$\psi(\mathbf{r}) = \frac{1}{\sqrt{V}} e^{i\mathbf{k} \cdot \mathbf{r}}. \quad (141)$$

$$\mathbf{k} = (k_x, k_y, k_z) = \left(\frac{n_x 2\pi}{L}, \frac{n_y 2\pi}{L}, \frac{n_z 2\pi}{L} \right), \quad (142)$$

$$E(\mathbf{k}) = \frac{\hbar^2 k^2}{2m_e} = \frac{\hbar^2}{2m_e} (k_x^2 + k_y^2 + k_z^2). \quad (143)$$

$$\frac{\hbar^2}{2m_e} \left(\frac{2\pi}{L} \right)^2, \quad (144)$$

$$\frac{N}{2} = \frac{4}{3} \pi n_{\max}^3, \quad (145)$$

$$n_{\max} = \left(\frac{3N}{8\pi} \right)^{1/3}. \quad (146)$$

$$E_{\max} = \frac{\hbar^2 k_{\max}^2}{2m_e} = \frac{\hbar^2}{2m_e} \left(\frac{2\pi}{L} \right)^2 n_{\max}^2. \quad (147)$$

$$E_F = \frac{\hbar^2}{2m_e} (3\pi^2 n)^{2/3}. \quad (148)$$

$$E(N) = \frac{\hbar^2}{2m_e} \left(\frac{3\pi^2 N}{V} \right)^{2/3}, \quad (149)$$

$$g(E) = \frac{dN}{dE} = \frac{V}{2\pi^2} \left(\frac{2m_e}{\hbar^2} \right)^{3/2} E^{1/2}. \quad (150)$$

$$f(E, T) = \frac{1}{e^{(E-\mu)/k_B T} + 1}, \quad (151)$$

$$N = \int_0^\infty g(E) f(E, T) dE. \quad (152)$$

$$\langle E \rangle = \frac{3}{2} k_B T g(E_F) k_B T \quad (153)$$

$$C = \frac{\partial \langle E \rangle}{\partial T} = 3k_B^2 T g(E_F). \quad (154)$$

$$C = \frac{\pi^2}{3} k_B^2 T g(E_F) \quad (155)$$

$$\frac{\kappa}{\sigma} = \frac{\pi^2}{3} \frac{k_B^2}{e^2} T = LT. \quad (156)$$

$$\phi_0(r) = \frac{1}{4\pi\epsilon_0} \frac{q}{r}, \quad (157)$$

$$\rho(r) = -e^2 (1/V) g(E_F) \phi(r). \quad (158)$$

$$\nabla^2 \phi(\mathbf{r}) = \frac{\partial^2 \phi(r)}{\partial r^2} + \frac{2}{r} \frac{\partial \phi(r)}{\partial r} = \frac{e^2}{V\epsilon_0} g(E_F) \phi(r). \quad (159)$$

$$\phi(r) = c \frac{1}{r} e^{-r/r_{\text{TF}}}, \quad (160)$$

$$r_{\text{TF}} = \sqrt{\frac{V\epsilon_0}{e^2 g(E_F)}}. \quad (161)$$

$$\phi(r) = \frac{1}{4\pi\epsilon_0} \frac{q}{r} e^{-r/r_{\text{TF}}}, \quad (162)$$

$$\psi_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} u_{\mathbf{k}}(\mathbf{r}), \quad (163)$$

$$u_{\mathbf{k}}(\mathbf{r}) = u_{\mathbf{k}}(\mathbf{r} + \mathbf{R}). \quad (164)$$

$$\psi_{\mathbf{k}}(\mathbf{r} + \mathbf{R}) = e^{i\mathbf{k}\cdot\mathbf{R}}\psi_{\mathbf{k}}(\mathbf{r}). \quad (165)$$

$$\psi(\mathbf{r}) = \sum_{\mathbf{k}} c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}, \quad (166)$$

$$U(\mathbf{r}) = \sum_{\mathbf{G}} U_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{r}}. \quad (167)$$

$$U_{-\mathbf{G}} = U_{\mathbf{G}}^*. \quad (168)$$

$$-\frac{\hbar^2 \nabla^2}{2m_e} \psi(\mathbf{r}) = \sum_{\mathbf{k}} \frac{\hbar^2 k^2}{2m_e} c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}}. \quad (169)$$

$$\begin{aligned} U(\mathbf{r})\psi(\mathbf{r}) &= \left(\sum_{\mathbf{G}} U_{\mathbf{G}} e^{i\mathbf{G}\cdot\mathbf{r}} \right) \left(\sum_{\mathbf{k}} c_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} \right) = \sum_{\mathbf{k}\mathbf{G}} U_{\mathbf{G}} c_{\mathbf{k}} e^{i(\mathbf{G}+\mathbf{k})\cdot\mathbf{r}} \\ &= \sum_{\mathbf{k}'\mathbf{G}} U_{\mathbf{G}} c_{\mathbf{k}'-\mathbf{G}} e^{i\mathbf{k}'\cdot\mathbf{r}}. \end{aligned} \quad (170)$$

$$\sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{r}} \left\{ \left(\frac{\hbar^2 k^2}{2m_e} - E \right) c_{\mathbf{k}} + \sum_{\mathbf{G}} U_{\mathbf{G}} c_{\mathbf{k}-\mathbf{G}} \right\} = 0. \quad (171)$$

$$\left(\frac{\hbar^2 k^2}{2m_e} - E \right) c_{\mathbf{k}} + \sum_{\mathbf{G}} U_{\mathbf{G}} c_{\mathbf{k}-\mathbf{G}} = 0. \quad (172)$$

$$\left(\frac{\hbar^2 (|\mathbf{k} - \mathbf{G}'|^2}{2m_e} - E \right) c_{\mathbf{k}-\mathbf{G}'} + \sum_{\mathbf{G}} U_{\mathbf{G}} c_{\mathbf{k}-\mathbf{G}'-\mathbf{G}} = 0. \quad (173)$$

$$\psi_{\mathbf{k}}(\mathbf{r}) = \sum_{\mathbf{G}} c_{\mathbf{k}-\mathbf{G}} e^{i(\mathbf{k}-\mathbf{G})\cdot\mathbf{r}}. \quad (174)$$

$$\psi_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} \left(\sum_{\mathbf{G}} c_{\mathbf{k}-\mathbf{G}} e^{-i\mathbf{G}\cdot\mathbf{r}} \right). \quad (175)$$

$$\psi_{\mathbf{k}+\mathbf{G}'}(\mathbf{r}) = \sum_{\mathbf{G}} c_{\mathbf{k}-\mathbf{G}+\mathbf{G}'} e^{i(\mathbf{k}-\mathbf{G}+\mathbf{G}')\cdot\mathbf{r}} = \sum_{\mathbf{G}''} c_{\mathbf{k}-\mathbf{G}''} e^{i(\mathbf{k}-\mathbf{G}'')\cdot\mathbf{r}}, \quad (176)$$

$$\psi_{\mathbf{k}+\mathbf{G}'} = \psi_{\mathbf{k}}(\mathbf{r}), \quad (177)$$

$$E(\mathbf{k} + \mathbf{G}') = E(\mathbf{k}). \quad (178)$$

$$U(x) = \sum_n U_n e^{ingx}, \quad (179)$$

$$\begin{aligned} \left(\frac{\hbar^2 (k-g)^2}{2m_e} - E \right) c_{k-g} + U c_k &= 0, \\ \left(\frac{\hbar^2 k^2}{2m_e} - E \right) c_k + U c_{k-g} + U c_{k+g} &= 0, \\ \left(\frac{\hbar^2 (k+g)^2}{2m_e} - E \right) c_{k+g} + U c_k &= 0. \end{aligned} \quad (180)$$

$$-i\hbar\nabla\psi_{\mathbf{k}}(\mathbf{r}) = \hbar\mathbf{k}\psi_{\mathbf{k}}(\mathbf{r}) - e^{i\mathbf{k}\cdot\mathbf{r}}i\hbar\nabla u_{\mathbf{k}}(\mathbf{r}). \quad (181)$$

$$\psi(+)\propto e^{i(\pi/a)x} + e^{-i(\pi/a)x} = 2\cos\left(\frac{\pi}{a}x\right), \quad (182)$$

$$\psi(-)\propto e^{i(\pi/a)x} - e^{-i(\pi/a)x} = 2i\sin\left(\frac{\pi}{a}x\right). \quad (183)$$

$$v_g = \frac{d\omega(k)}{dk} = \frac{1}{\hbar} \frac{dE(k)}{dk}. \quad (184)$$

$$H_{at} = -\frac{\hbar^2\nabla^2}{2m_e} + V_{at}(\mathbf{r}), \quad (185)$$

$$H_{sol} = -\frac{\hbar^2\nabla^2}{2m_e} + \sum_{\mathbf{R}} V_{at}(\mathbf{r} - \mathbf{R}) = -\frac{\hbar^2\nabla^2}{2m_e} + V_{at}(\mathbf{r}) + \sum_{\mathbf{R}\neq 0} V_{at}(\mathbf{r} - \mathbf{R}). \quad (186)$$

$$H_{sol} = -\frac{\hbar^2\nabla^2}{2m_e} + V_{at}(\mathbf{r}) + v(\mathbf{r}) = H_{at} + v(\mathbf{r}), \quad (187)$$

$$v(\mathbf{r}) = \sum_{\mathbf{R}\neq 0} V_{at}(\mathbf{r} - \mathbf{R}). \quad (188)$$

$$\int \phi_n^*(\mathbf{r})H_{sol}\phi_n(\mathbf{r})d\mathbf{r} = E_n + \int \phi_n^*(\mathbf{r})v(\mathbf{r})\phi_n(\mathbf{r})d\mathbf{r} = E_n - \beta, \quad (189)$$

$$\psi_{\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{N}} \sum_{\mathbf{R}} c_{\mathbf{k},\mathbf{R}}\phi_n(\mathbf{r} - \mathbf{R}). \quad (190)$$

$$\psi_{\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{N}} \sum_{\mathbf{R}} e^{i\mathbf{k}\cdot\mathbf{R}}\phi_n(\mathbf{r} - \mathbf{R}), \quad (191)$$

$$\begin{aligned} \psi_{\mathbf{k}}(\mathbf{r} + \mathbf{R}') &= \frac{1}{\sqrt{N}} \sum_{\mathbf{R}} e^{i\mathbf{k}\cdot\mathbf{R}}\phi_n(\mathbf{r} - \mathbf{R} + \mathbf{R}') = \\ &= \frac{1}{\sqrt{N}} e^{i\mathbf{k}\cdot\mathbf{R}'} \sum_{\mathbf{R}} e^{i\mathbf{k}\cdot(\mathbf{R}-\mathbf{R}')} \phi_n(\mathbf{r} - (\mathbf{R} - \mathbf{R}')) = \\ &= \frac{1}{\sqrt{N}} e^{i\mathbf{k}\cdot\mathbf{R}'} \sum_{\mathbf{R}''} e^{i\mathbf{k}\cdot\mathbf{R}''} \phi_n(\mathbf{r} - \mathbf{R}'') = e^{i\mathbf{k}\cdot\mathbf{R}'} \psi_{\mathbf{k}}(\mathbf{r}), \end{aligned} \quad (192)$$

$$E(\mathbf{k}) = \int \psi_{\mathbf{k}}^*(\mathbf{r})H_{sol}\psi_{\mathbf{k}}(\mathbf{r})d\mathbf{r} = \frac{1}{N} \sum_{\mathbf{R},\mathbf{R}'} e^{i\mathbf{k}\cdot(\mathbf{R}-\mathbf{R}')} \int \phi_n^*(\mathbf{r} - \mathbf{R}')H_{sol}\phi_n(\mathbf{r} - \mathbf{R})d\mathbf{r} \quad (193)$$

$$E(\mathbf{k}) = \sum_{\mathbf{R}} e^{i\mathbf{k}\cdot\mathbf{R}} \int \phi_n^*(\mathbf{r})H_{sol}\phi_n(\mathbf{r} - \mathbf{R})d\mathbf{r}. \quad (194)$$

$$E(\mathbf{k}) = E_n - \beta + \sum_{\mathbf{R} \neq 0} e^{i\mathbf{k} \cdot \mathbf{R}} \int \phi_n^*(\mathbf{r}) H_{sol} \phi_n(\mathbf{r} - \mathbf{R}) d\mathbf{r}. \quad (195)$$

$$\int \phi_n^*(\mathbf{r}) H_{sol} \phi_n(\mathbf{r} - \mathbf{R}) d\mathbf{r} = E_n \int \phi_n^*(\mathbf{r}) \phi_n(\mathbf{r} - \mathbf{R}) d\mathbf{r} + \int \phi_n^*(\mathbf{r}) v(\mathbf{r}) \phi_n(\mathbf{r} - \mathbf{R}) d\mathbf{r} \quad (196)$$

$$\gamma(\mathbf{R}) = - \int \phi_n^*(\mathbf{r}) v(\mathbf{r}) \phi_n(\mathbf{r} - \mathbf{R}) d\mathbf{r}, \quad (197)$$

$$E(\mathbf{k}) = E_n - \beta - \sum_{\mathbf{R} \neq 0} \gamma(\mathbf{R}) e^{i\mathbf{k} \cdot \mathbf{R}}. \quad (198)$$

$$E_s(\mathbf{k}) = E_s - \beta_s - \gamma_s (e^{ika} + e^{-ika}) = E_s - \beta_s - 2\gamma_s \cos ka, \quad (199)$$

$$dE = -e\mathcal{E} v_g dt. \quad (200)$$

$$\frac{dE}{dt} = \frac{dE}{dk} \frac{dk}{dt}, \quad (201)$$

$$\hbar \frac{dk}{dt} = -e\mathcal{E}. \quad (202)$$

$$a = \frac{dv_g}{dt} = \frac{1}{\hbar} \frac{d}{dt} \frac{dE(k)}{dk} = \frac{1}{\hbar} \frac{d^2 E(k)}{dk^2} \frac{dk}{dt}. \quad (203)$$

$$a = -\frac{1}{\hbar^2} \frac{d^2 E(k)}{dk^2} e\mathcal{E}. \quad (204)$$

$$m^* = \hbar^2 \left(\frac{d^2 E(k)}{dk^2} \right)^{-1}. \quad (205)$$

0.7 Semiconductors

$$n = \frac{1}{V} \int_{E_C}^{\infty} g_C(E) f(E, T) dE, \quad (206)$$

$$p = \frac{1}{V} \int_{-\infty}^{E_V} g_V(E) [1 - f(E, T)] dE, \quad (207)$$

$$E = E_g + \frac{\hbar^2 k^2}{2m_e^*}. \quad (208)$$

$$g_C(E) = \frac{V}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2} \right)^{3/2} (E - E_g)^{1/2}. \quad (209)$$

$$E = -\frac{\hbar^2 k^2}{2m_h^*} \quad (210)$$

$$g_V(E) = \frac{V}{2\pi^2} \left(\frac{2m_h^*}{\hbar^2} \right)^{3/2} (-E)^{1/2}. \quad (211)$$

$$f(E, T) = \frac{1}{e^{(E-\mu)/k_B T} + 1} \approx e^{-(E-\mu)/k_B T}. \quad (212)$$

$$1 - f(E, T) = 1 - \frac{1}{e^{(E-\mu)/k_B T} + 1} \approx e^{(E-\mu)/k_B T} \quad (213)$$

$$\begin{aligned} n &= \frac{1}{V} \int_{E_g}^{\infty} \frac{V}{2\pi^2} \left(\frac{2m_e^*}{\hbar^2} \right)^{3/2} (E - E_g)^{1/2} e^{-(E-\mu)/k_B T} dE \\ &= \frac{(2m_e^*)^{3/2}}{2\pi^2 \hbar^3} e^{\mu/k_B T} \int_{E_g}^{\infty} (E - E_g)^{1/2} e^{-E/k_B T} dE. \end{aligned} \quad (214)$$

$$n = \frac{(2m_e^*)^{3/2}}{2\pi^2 \hbar^3} (k_B T)^{3/2} e^{-(E_g-\mu)/k_B T} \int_0^{\infty} X_g^{1/2} e^{-X_g} dX_g. \quad (215)$$

$$n = \frac{1}{\sqrt{2}} \left(\frac{m_e^* k_B T}{\pi \hbar^2} \right)^{3/2} e^{-(E_g-\mu)/k_B T} = N_{\text{eff}}^C e^{-(E_g-\mu)/k_B T}, \quad (216)$$

$$p = \frac{1}{\sqrt{2}} \left(\frac{m_h^* k_B T}{\pi \hbar^2} \right)^{3/2} e^{-\mu/k_B T} = N_{\text{eff}}^V e^{-\mu/k_B T}. \quad (217)$$

$$np = 4 \left(\frac{k_B T}{2\pi \hbar^2} \right)^3 (m_e^* m_h^*)^{3/2} e^{-E_g/k_B T}, \quad (218)$$

$$n_i = p_i = \sqrt{np} = 2 \left(\frac{k_B T}{2\pi \hbar^2} \right)^{3/2} (m_e^* m_h^*)^{3/4} e^{-E_g/2k_B T}. \quad (219)$$

$$\mu = \frac{E_g}{2} + \frac{3}{4} k_B T \ln \left(\frac{m_h^*}{m_e^*} \right). \quad (220)$$

$$\omega_c = \frac{Be}{m_e^*}. \quad (221)$$

$$E_n = -\frac{m_e e^4}{8\epsilon_0^2 \hbar^2} \frac{1}{n^2}. \quad (222)$$

$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{m_e e^2}. \quad (223)$$

$$\sigma = e(n\mu_e + p\mu_h), \quad (224)$$

$$\frac{d^2 U}{dx^2} = -\frac{\rho}{\epsilon\epsilon_0}, \quad (225)$$

$$\left. \frac{dU}{dx} \right|_{x=-d_p, d_n} = 0. \quad (226)$$

$$\Delta U = \frac{e}{2\epsilon\epsilon_0} \left(N_d d_n^2 + N_a d_p^2 \right). \quad (227)$$

$$d_p = \left(\frac{\Delta U 2\epsilon\epsilon_0}{e N_a} \frac{N_d}{N_a + N_d} \right)^{1/2}, \quad d_n = \left(\frac{\Delta U 2\epsilon\epsilon_0}{e N_d} \frac{N_a}{N_a + N_d} \right)^{1/2}. \quad (228)$$

$$n_p = N_{\text{eff}}^C e^{(\mu - E_g - e\Delta U)/k_B T}, \quad p_p = N_{\text{eff}}^V e^{(e\Delta U - \mu)/k_B T}, \quad (229)$$

$$n_n = N_{\text{eff}}^C e^{(\mu - E_g)/k_B T}, \quad p_n = N_{\text{eff}}^V e^{-\mu/k_B T}. \quad (230)$$

$$|I_{\text{diffusion}}| = |I_{\text{drift}}| = |I_0| = C e^{(\mu - E_g - e\Delta U)/k_B T}, \quad (231)$$

$$|I_{\text{diffusion}}| = C e^{((\mu + eV) - E_g - e\Delta U)/k_B T}. \quad (232)$$

$$I = I_{\text{diffusion}} - I_{\text{drift}} = I_0 \left(e^{eV/k_B T} - 1 \right). \quad (233)$$

$$|I_{\text{diffusion}}| = C e^{((\mu - eV) - E_g - e\Delta U)/k_B T}, \quad (234)$$

$$I = I_{\text{diffusion}} - I_{\text{drift}} = I_0 \left(e^{-eV/k_B T} - 1 \right). \quad (235)$$

0.8 Magnetism

$$\oint \mathbf{B} d\mathbf{a} = 0, \quad \text{div} \mathbf{B} = 0 \quad (236)$$

$$\mathbf{B} = \mu_0 \mathbf{H}, \quad (237)$$

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) = \mathbf{B}_0 + \mu_0 \mathbf{M}, \quad (238)$$

$$\mathbf{M} = \mu \frac{N}{V}. \quad (239)$$

$$\mu_0 \mathbf{M} = \chi_m \mathbf{B}_0, \quad (240)$$

$$U = -V \int_0^{B_0} M dB'_0 = -V \int_0^{B_0} \frac{\chi_m}{\mu_0} B'_0 dB'_0 = -V \frac{\chi_m}{2\mu_0} B_0^2, \quad (241)$$

$$\mathbf{B} = \text{curl} \mathbf{A}. \quad (242)$$

$$\mathbf{p} \rightarrow \mathbf{p} - q\mathbf{A}. \quad (243)$$

$$\mathbf{A} = -\frac{1}{2} \mathbf{r} \times \mathbf{B}_0, \quad (244)$$

$$\frac{\mathbf{p}^2}{2m_e} \rightarrow \frac{1}{2m_e} (\mathbf{p} + e\mathbf{A})^2 = \frac{1}{2m_e} \left(\mathbf{p} - e \frac{\mathbf{r} \times \mathbf{B}_0}{2} \right)^2. \quad (245)$$

$$H'_{\text{kin}} = \frac{1}{2m_e} \left(\mathbf{p}^2 + e\mathbf{B}_0 \cdot (\mathbf{r} \times \mathbf{p}) + \frac{e^2}{4} (\mathbf{r} \times \mathbf{B}_0)^2 \right), \quad (246)$$

$$H'_{\text{kin}} = H_{\text{kin}} + H' = \frac{\mathbf{p}^2}{2m_e} + \frac{e}{2m_e} B_0 (\mathbf{r} \times \mathbf{p})_z + \frac{e^2}{8m_e} B_0^2 (x^2 + y^2). \quad (247)$$

$$E' = \frac{e}{2m_e} B_0 \langle \psi | (\mathbf{r} \times \mathbf{p})_z | \psi \rangle + \frac{e^2}{8m_e} B_0^2 \langle \psi | (x^2 + y^2) | \psi \rangle. \quad (248)$$

$$g_e m_s \frac{e\hbar}{2m_e} B_0 = g_e m_s \mu_B B_0, \quad (249)$$

$$\boldsymbol{\mu} = -\frac{e\hbar}{2m_e} \mathbf{L} = -\mu_B \mathbf{L}. \quad (250)$$

$$\mu_l = -\frac{em_l \hbar}{2m_e} = -m_l \mu_B. \quad (251)$$

$$\boldsymbol{\mu} = -g_e \mu_B \mathbf{S}, \quad (252)$$

$$\mu_s = -g_e m_s \mu_B, \quad (253)$$

$$\mu_J = -g m_J \mu_B, \quad (254)$$

$$L = \sum m_l \quad S = \sum m_s. \quad (255)$$

$$g_J = \frac{3J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}. \quad (256)$$

$$\mu = -\frac{\partial E'}{\partial B_0} = -\frac{e^2}{4m_e} B_0 \langle \psi | (x^2 + y^2) | \psi \rangle. \quad (257)$$

$$\mu = -\frac{Z e^2}{6m_e} r_a^2 B_0. \quad (258)$$

$$\chi_m = \mu_0 \frac{M}{B_0} = -\frac{\mu_0 Z N e^2}{6V m_e} r_a^2 \quad (259)$$

$$\chi_m = -\frac{1}{3V} \mu_B^2 \mu_0 g(E_F) \left(\frac{m_e}{m^*} \right)^2. \quad (260)$$

$$\bar{\mu} = \frac{1}{Z} \sum_{m_J=-J}^J g_J \mu_B m_J e^{g_J \mu_B m_J B_0 / k_B T}, \quad (261)$$

$$Z = \sum_{m_J=-J}^J e^{-g_J \mu_B m_J B_0 / k_B T}. \quad (262)$$

$$\chi_m = \frac{C}{T}, \quad (263)$$

$$C = \frac{\mu_0 N g_J^2 \mu_B^2 J(J+1)}{3V k_B}. \quad (264)$$

$$N_{\downarrow \downarrow B_0} - N_{\downarrow \uparrow B_0} = g(E_F) \mu_B B_0, \quad (265)$$

$$M = \frac{1}{V} (N_{\downarrow \downarrow B_0} - N_{\downarrow \uparrow B_0}) \mu_B = \frac{1}{V} g(E_F) \mu_B^2 B_0, \quad (266)$$

$$\chi_m = \frac{1}{V} \mu_0 \mu_B^2 g(E_F), \quad (267)$$

$$E_{\uparrow \uparrow} - E_{\uparrow \downarrow} = -2X. \quad (268)$$

$$H = -2X \mathbf{S}_1 \cdot \mathbf{S}_2. \quad (269)$$

$$H = -\sum_{i \neq j} \sum X_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + g_e \mu_B \mathbf{B}_0 \cdot \sum_i \mathbf{S}_i, \quad (270)$$

$$H = -X \sum_i \sum_{nn} \mathbf{S}_i \cdot \mathbf{S}_{nn} + g_e \mu_B \mathbf{B}_0 \cdot \sum_i \mathbf{S}_i, \quad (271)$$

$$H = \sum_i \mathbf{S}_i \cdot \left(-\sum_{nn} X \langle \mathbf{S} \rangle + g_e \mu_B \mathbf{B}_0 \right) = \sum_i \mathbf{S}_i \cdot (-n_{nn} X \langle \mathbf{S} \rangle + g_e \mu_B \mathbf{B}_0), \quad (272)$$

$$\mathbf{M} = -g_e \mu_B \langle \mathbf{S} \rangle \frac{N}{V}, \quad (273)$$

$$H = \sum_i \mathbf{S}_i \cdot \left(\frac{n_{nn} X V}{g_e \mu_B N} \mathbf{M} + g_e \mu_B \mathbf{B}_0 \right) = g_e \mu_B \sum_i \mathbf{S}_i \cdot (\mathbf{B}_W + \mathbf{B}_0), \quad (274)$$

$$\mathbf{B}_W = \mathbf{M} \frac{n_{nn} X V}{g_e^2 \mu_B^2 N}. \quad (275)$$

$$M(T) = \frac{\mu_B N}{V} \frac{e^x - e^{-x}}{e^{-x} + e^x} = M(0) \tanh(x), \quad (276)$$

$$\frac{M(T)}{M(0)} = \tanh \left(\frac{M(T)}{M(0)} \frac{\Theta_C}{T} \right), \quad (277)$$

$$\Theta_C = \frac{n_{nn} X}{g_e^2 k_B}. \quad (278)$$

$$\chi_m = \frac{C}{T - \Theta_c}. \quad (279)$$

0.9 Dielectrics

$$\mathbf{P} = \chi_e \epsilon_0 \mathcal{E}, \quad (280)$$

$$\mathbf{P} = \frac{N}{V} \mathbf{p} = \frac{N}{V} q \boldsymbol{\delta}. \quad (281)$$

$$\chi_e = \epsilon - 1. \quad (282)$$

$$\mathbf{p} = \alpha \mathcal{E}, \quad (283)$$

$$\mathbf{P} = (\epsilon - 1) \epsilon_0 \mathcal{E} = \frac{N}{V} \mathbf{p} = \frac{N}{V} \alpha \mathcal{E}, \quad (284)$$

$$\alpha = \frac{(\epsilon - 1) \epsilon_0 V}{N}. \quad (285)$$

$$\mathcal{E}_{\text{loc}} = \frac{1}{3} (\epsilon + 2) \mathcal{E}, \quad (286)$$

$$\mathbf{P} = \frac{N}{V} \alpha \mathcal{E}_{\text{loc}} = \frac{N \alpha}{3V} (\epsilon + 2) \mathcal{E}. \quad (287)$$

$$\alpha = \frac{\epsilon - 1}{\epsilon + 2} \frac{3 \epsilon_0 V}{N}. \quad (288)$$

$$\frac{d^2 x}{dt^2} + \eta \frac{dx}{dt} + \omega_0^2 x = \frac{e \mathcal{E}_0}{M} e^{-i\omega t}. \quad (289)$$

$$x(t) = A e^{-i\omega t}, \quad (290)$$

$$A = \frac{e \mathcal{E}_0}{M} \frac{1}{\omega_0^2 - \omega^2 - i\eta\omega}. \quad (291)$$

$$A = \frac{e \mathcal{E}_0}{M} \left(\frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \eta^2 \omega^2} + \frac{i\eta\omega}{(\omega_0^2 - \omega^2)^2 + \eta^2 \omega^2} \right). \quad (292)$$

$$P(t) = P_i(t) + P_e(t) = \frac{N}{V} e A e^{-i\omega t} + \frac{N}{V} \alpha \mathcal{E}_0 e^{-i\omega t}. \quad (293)$$

$$\epsilon = \frac{P(t)}{\epsilon_0 \mathcal{E}_0 e^{-i\omega t}} + 1 = \frac{N e A}{V \epsilon_0 \mathcal{E}_0} + \frac{N \alpha}{V \epsilon_0} + 1. \quad (294)$$

$$\epsilon_{\text{opt}} = \frac{N\alpha}{V\epsilon_0} + 1, \quad (295)$$

$$\epsilon(\omega) = \frac{NeA}{V\epsilon_0\mathcal{E}_0} + \epsilon_{\text{opt}}. \quad (296)$$

$$\epsilon_r(\omega) = \frac{Ne^2}{V\epsilon_0M} \frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \eta^2\omega^2} + \epsilon_{\text{opt}} \quad (297)$$

$$\epsilon_i(\omega) = \frac{Ne^2}{V\epsilon_0M} \frac{\eta\omega}{(\omega_0^2 - \omega^2)^2 + \eta^2\omega^2}. \quad (298)$$

$$p(t) = j(t)\mathcal{E}(t), \quad (299)$$

$$j(t) = -\frac{\partial D}{\partial t} = -\frac{\partial}{\partial t}\epsilon\epsilon_0\mathcal{E}(t) = \epsilon_0\mathcal{E}(t)\left(i\omega\epsilon_r - \omega\epsilon_i\right). \quad (300)$$

$$\bar{p} = \frac{1}{T} \int_0^T \mathcal{E}(t)j(t)dt, \quad (301)$$

$$\frac{1}{2}\epsilon_0\epsilon_i\mathcal{E}_0^2. \quad (302)$$

$$\epsilon_i(h\nu) \propto \sum_{\mathbf{k}} M^2\delta(E_C(\mathbf{k}) - E_V(\mathbf{k}) - h\nu), \quad (303)$$

0.10 Superconductivity

$$B_C(T) = B_C(0) \left[1 - \left(\frac{T}{T_C} \right)^2 \right]. \quad (304)$$

$$\oint \mathcal{E}d\mathbf{l} = -\frac{d\Phi_B}{dt}, \quad (305)$$

$$\frac{\partial \mathbf{j}}{\partial t} = \frac{n_s q^2}{m_s} \mathcal{E}, \quad (306)$$

$$\frac{\partial}{\partial t} \left(\frac{m_s}{n_s q^2} \text{curl} \mathbf{j} + \mathbf{B} \right) = 0. \quad (307)$$

$$\frac{\partial}{\partial t} \left(\int \frac{m_s}{n_s q^2} \text{curl} \mathbf{j} d\mathbf{A} + \int \mathbf{B} d\mathbf{A} \right) = \frac{\partial}{\partial t} \left(\oint \frac{m_s}{n_s q^2} \mathbf{j} d\mathbf{l} + \int \mathbf{B} d\mathbf{A} \right) = 0. \quad (308)$$

$$\frac{m_s}{n_s q^2} \text{curl} \mathbf{j} + \mathbf{B} = 0. \quad (309)$$

$$\text{curl} \mathbf{B} = \mu_0 \mathbf{j}. \quad (310)$$

$$\text{curl} \text{curl} \mathbf{B} = \mu_0 \text{curl} \mathbf{j} = -\frac{\mu_0 n_s q^2}{m_s} \mathbf{B}. \quad (311)$$

$$\Delta \mathbf{B} = \frac{\mu_0 n_s q^2}{m_s} \mathbf{B}. \quad (312)$$

$$\Delta \mathbf{j} = \frac{\mu_0 n_s q^2}{m_s} \mathbf{j}. \quad (313)$$

$$\lambda_L = \sqrt{m_s / \mu_0 n_s q^2}. \quad (314)$$

$$\Psi(\mathbf{r}) = \Psi_0(\mathbf{r})e^{i\phi(\mathbf{r})}, \quad (315)$$

$$T_C = 1.13\Theta_D \exp \frac{-1}{g(E_F)V}, \quad (316)$$

$$\oint \frac{\mathbf{p}}{h} d\mathbf{r} = n, \quad (317)$$

$$\oint \mathbf{p} - q\mathbf{A} d\mathbf{r} = nh. \quad (318)$$

$$\frac{m_s}{n_s q} \oint \mathbf{j} d\mathbf{r} - q \oint \mathbf{A} d\mathbf{r} = nh. \quad (319)$$

$$\oint \mathbf{A} d\mathbf{r} = \int \text{curl} \mathbf{A} d\mathbf{a} = \int \mathbf{B} d\mathbf{a} = \Phi_B, \quad (320)$$

$$\frac{m_s}{n_s q^2} \oint \mathbf{j} d\mathbf{r} - \Phi_B = n \frac{h}{q}. \quad (321)$$

$$\Phi_B = n \frac{h}{q}. \quad (322)$$

0.11 Finite Solids and Nanostructures

$$\Psi(\mathbf{r}) = \Psi(z)\Psi(x, y). \quad (323)$$

$$E_{xy} = \frac{\hbar^2 \mathbf{k}_{xy}^2}{2m_e}, \quad (324)$$

$$\Psi(z) = Ae^{ik_z z} + Be^{-ik_z z}, \quad (325)$$

$$k_z d = n\pi, \quad n = 1, 2, 3, \dots \quad (326)$$

$$E_z = \frac{\hbar^2 k_z^2}{2m_e}, \quad (327)$$

$$2k_z d + \Phi_i + \Phi_v = 2\pi n, \quad n = 1, 2, 3, \dots \quad (328)$$

$$E_{\min} = E_g + \frac{\hbar^2 \pi^2}{2\mu r^2} - \frac{1.8e^2}{4\pi\epsilon_0\epsilon r} \quad (329)$$

$$\psi_{\mathbf{k}}(\mathbf{r}) = e^{i\mathbf{k}\mathbf{r}} u_{\mathbf{k}}(\mathbf{r}). \quad (330)$$

$$\psi_{\mathbf{k}}(\mathbf{r}) = e^{\Im(k_z)z} e^{i\mathbf{k}'\cdot\mathbf{r}} u_{\mathbf{k}}(\mathbf{r}), \quad (331)$$

$$\Delta E \approx V\mu_0 M^2. \quad (332)$$