0.1 Crystal Structures

$$\mathbf{R} = m\mathbf{a_1} + n\mathbf{a_2}, \tag{1}$$

$$\mathbf{R} = m\mathbf{a_1} + n\mathbf{a_2} + o\mathbf{a_3}. \tag{2}$$

$$n\lambda = 2d\sin\theta. \tag{3}$$

$$\boldsymbol{\mathcal{E}}(\mathbf{r},t) = \boldsymbol{\mathcal{E}}_0\,\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}-\mathrm{i}\omega t}. \tag{4}$$

$$I(\mathbf{r}) = \left|\boldsymbol{\mathcal{E}}_0\,\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}-\mathrm{i}\omega t}\right|^2 = \left|\boldsymbol{\mathcal{E}}_0\right|^2. \tag{5}$$

$$\boldsymbol{\mathcal{E}}(\mathbf{r},t) = \boldsymbol{\mathcal{E}}_0\,\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{R})-\mathrm{i}\omega t}. \tag{6}$$

$$\boldsymbol{\mathcal{E}}(\mathbf{r},t) \propto \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{R})}\,\mathrm{e}^{-\mathrm{i}\omega t}. \tag{7}$$

$$\boldsymbol{\mathcal{E}}(\mathbf{R}',t) \propto \boldsymbol{\mathcal{E}}(\mathbf{r},t)\rho(\mathbf{r})\,\mathrm{e}^{\mathrm{i}\mathbf{k}'\cdot(\mathbf{R}'-\mathbf{r})}. \tag{8}$$

$$\boldsymbol{\mathcal{E}}(\mathbf{R}',t) \propto \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{R})}\rho(\mathbf{r})\,\mathrm{e}^{\mathrm{i}\mathbf{k}'\cdot(\mathbf{R}'-\mathbf{r})}\,\mathrm{e}^{-\mathrm{i}\omega t} = \mathrm{e}^{\mathrm{i}(\mathbf{k}'\cdot\mathbf{R}'-\mathbf{k}\cdot\mathbf{R})}\rho(\mathbf{r})\,\mathrm{e}^{\mathrm{i}(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}}\,\mathrm{e}^{-\mathrm{i}\omega t}. \tag{9}$$

$$\boldsymbol{\mathcal{E}}(\mathbf{R}',t) \propto \mathrm{e}^{-\mathrm{i}\omega t}\int_V \rho(\mathbf{r})\,\mathrm{e}^{\mathrm{i}(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}}\,\mathrm{d}V. \tag{10}$$

$$I(\mathbf{K}) \propto \left|\mathrm{e}^{-\mathrm{i}\omega t}\int_V \rho(\mathbf{r})\,\mathrm{e}^{\mathrm{i}(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}}\,\mathrm{d}V\right|^2 = \left|\int_V \rho(\mathbf{r})\,\mathrm{e}^{-\mathrm{i}\mathbf{K}\cdot\mathbf{r}}\,\mathrm{d}V\right|^2, \tag{11}$$

$$\mathbf{R} = m\mathbf{a_1} + n\mathbf{a_2} + o\mathbf{a_3}, \tag{12}$$

$$\mathbf{R}\cdot\mathbf{G} = 2\pi l, \tag{13}$$

$$\mathrm{e}^{\mathrm{i}\mathbf{G}\cdot\mathbf{R}} = 1. \tag{14}$$

$$\mathbf{G} = m'\,\mathbf{b_1} + n'\,\mathbf{b_2} + o'\,\mathbf{b_3}, \tag{15}$$

$$\mathbf{b}_1 = 2\pi\frac{\mathbf{a}_2\times\mathbf{a}_3}{\mathbf{a}_1\cdot(\mathbf{a}_2\times\mathbf{a}_3)}, \quad \mathbf{b}_2 = 2\pi\frac{\mathbf{a}_3\times\mathbf{a}_1}{\mathbf{a}_1\cdot(\mathbf{a}_2\times\mathbf{a}_3)}, \quad \mathbf{b}_3 = 2\pi\frac{\mathbf{a}_1\times\mathbf{a}_2}{\mathbf{a}_1\cdot(\mathbf{a}_2\times\mathbf{a}_3)}. \tag{16}$$

$$\mathbf{a}_i\cdot\mathbf{b}_j = 2\pi\,\delta_{ij}, \tag{17}$$

$$\mathrm{e}^{\mathrm{i}\mathbf{G}\cdot\mathbf{r}} = \mathrm{e}^{\mathrm{i}\mathbf{G}\cdot\mathbf{r}}\,\mathrm{e}^{\mathrm{i}\mathbf{G}\cdot\mathbf{R}} = \mathrm{e}^{\mathrm{i}\mathbf{G}\cdot(\mathbf{r}+\mathbf{R})}. \tag{18}$$

$$\rho(x) = C + \sum_{n=1}^{\infty}\left\{C_n\cos(x\,2\pi\,n/a) + S_n\sin(x\,2\pi\,n/a)\right\} \tag{19}$$

$$\rho(x) = \sum_{n=-\infty}^{\infty}\rho_n\,\mathrm{e}^{\mathrm{i}xn\,2\pi/a}. \tag{20}$$

$$\rho^*_{-n} = \rho_n, \tag{21}$$

$$g = n\frac{2\pi}{a}, \tag{22}$$

$$\rho(\mathbf{r}) = \sum_{\mathbf{G}}\rho_{\mathbf{G}}\,\mathrm{e}^{\mathrm{i}\mathbf{G}\cdot\mathbf{r}}, \tag{23}$$

$$I(\mathbf{K}) \propto \left|\sum_{\mathbf{G}}\rho_{\mathbf{G}}\int_V \mathrm{e}^{\mathrm{i}(\mathbf{G}-\mathbf{K})\cdot\mathbf{r}}\,\mathrm{d}V\right|^2. \tag{24}$$

$$\mathbf{K} = \mathbf{k}' - \mathbf{k} = \mathbf{G}, \tag{25}$$

$$I(\mathbf{G}) \propto \left| \int_V \rho(\mathbf{r})\, \mathrm{e}^{-\mathrm{i}\mathbf{G}\cdot\mathbf{r}}\, \mathrm{d}V \right|^2 . \tag{26}$$

$$I(\mathbf{G}) \propto \left| \sum_{\mathbf{R}} \int_{V_\text{cell}} \rho(\mathbf{r}+\mathbf{R})\, \mathrm{e}^{-\mathrm{i}\mathbf{G}\cdot(\mathbf{r}+\mathbf{R})}\, \mathrm{d}V \right|^2 = \left| N \int_{V_\text{cell}} \rho(\mathbf{r})\, \mathrm{e}^{-\mathrm{i}\mathbf{G}\cdot\mathbf{r}}\, \mathrm{d}V \right|^2 , \tag{27}$$

$$\rho(\mathbf{r}) = \sum_i \rho_i(\mathbf{r}-\mathbf{r}_i), \tag{28}$$

$$\int_{V_\text{cell}} \rho(\mathbf{r})\, \mathrm{e}^{-\mathrm{i}\,\mathbf{G}\cdot\mathbf{r}}\, \mathrm{d}V = \sum_i \mathrm{e}^{-\mathrm{i}\,\mathbf{G}\cdot\mathbf{r}_i} \int_{V_\text{atom}} \rho_i(\mathbf{r}')\, \mathrm{e}^{-\mathrm{i}\mathbf{G}\cdot\mathbf{r}'}\, \mathrm{d}V', \tag{29}$$

$$k'_\perp - k_\perp = 2k_\perp = 2\frac{2\pi}{\lambda}\sin\Theta = G_\perp, \tag{30}$$

0.2 Bonding in Solids

$$\phi(r) = \frac{A}{r^n} - \frac{B}{r^m}, \tag{31}$$

$$E_\text{Na} = -1.748\frac{e^2}{4\pi\varepsilon_0 a} = -\alpha\frac{e^2}{4\pi\varepsilon_0 a}. \tag{32}$$

$$\begin{aligned} H = &-\frac{\hbar^2\nabla_1^2}{2m_\mathrm{e}} - \frac{\hbar^2\nabla_2^2}{2m_\mathrm{e}} + \frac{e^2}{4\pi\varepsilon_0}\left\{\frac{1}{R} + \frac{1}{|\mathbf{r}_1-\mathbf{r}_2|}\right. \\ &\left.-\frac{1}{|\mathbf{r}_1-\mathbf{R}_\mathrm{A}|} - \frac{1}{|\mathbf{r}_2-\mathbf{R}_\mathrm{B}|} - \frac{1}{|\mathbf{r}_2-\mathbf{R}_\mathrm{A}|} - \frac{1}{|\mathbf{r}_1-\mathbf{R}_\mathrm{B}|}\right\}, \end{aligned} \tag{33}$$

$$H' = -\frac{\hbar^2\nabla^2}{2m_\mathrm{e}} + \frac{e^2}{4\pi\varepsilon_0}\left\{\frac{1}{R} - \frac{1}{|\mathbf{r}-\mathbf{R}_\mathrm{A}|} - \frac{1}{|\mathbf{r}-\mathbf{R}_\mathrm{B}|}\right\}. \tag{34}$$

$$H'\Psi(\mathbf{r}) = H'\big(c_1\phi_\mathrm{A}(\mathbf{r}) + c_2\phi_\mathrm{B}(\mathbf{r})\big) = E'\big(c_1\phi_\mathrm{A}(\mathbf{r}) + c_2\phi_\mathrm{B}(\mathbf{r})\big), \tag{35}$$

$$\begin{aligned} c_1H'_\mathrm{AA} + c_2H'_\mathrm{AB} &= c_1E' + c_2SE' \\ c_1H'_\mathrm{BA} + c_2H'_\mathrm{BB} &= c_2E' + c_1SE', \end{aligned} \tag{36}$$

$$\begin{aligned} c_1(H'_\mathrm{AA} - E') + c_2(H'_\mathrm{AB} - SE') &= 0 \\ c_1(H'_\mathrm{AB} - SE') + c_2(H'_\mathrm{AA} - E') &= 0. \end{aligned} \tag{37}$$

$$E'_\pm = \frac{H'_\mathrm{AA} \pm H'_\mathrm{AB}}{1 \pm S}. \tag{38}$$

$$\Psi_{\uparrow\downarrow}(\mathbf{r}_1,\mathbf{r}_2) \propto \phi_\mathrm{A}(\mathbf{r}_1)\phi_\mathrm{B}(\mathbf{r}_2) + \phi_\mathrm{A}(\mathbf{r}_2)\phi_\mathrm{B}(\mathbf{r}_1), \tag{39}$$

$$\Psi_{\uparrow\uparrow}(\mathbf{r}_1,\mathbf{r}_2) \propto \phi_\mathrm{A}(\mathbf{r}_1)\phi_\mathrm{B}(\mathbf{r}_2) - \phi_\mathrm{A}(\mathbf{r}_2)\phi_\mathrm{B}(\mathbf{r}_1). \tag{40}$$

$$E = \frac{\int \Psi^*(\mathbf{r}_1,\mathbf{r}_2)H\Psi(\mathbf{r}_1,\mathbf{r}_2)\,\mathrm{d}\mathbf{r}_1\,\mathrm{d}\mathbf{r}_2}{\int \Psi^*(\mathbf{r}_1,\mathbf{r}_2)\Psi(\mathbf{r}_1,\mathbf{r}_2)\,\mathrm{d}\mathbf{r}_1\,\mathrm{d}\mathbf{r}_2}. \tag{41}$$

$$E_\text{singlet} = 2E_0 + \Delta E_{\uparrow\downarrow}, \tag{42}$$

$$E_\text{triplet} = 2E_0 + \Delta E_{\uparrow\uparrow}. \tag{43}$$

$$E = 2E_0 + C \pm X, \tag{44}$$

0.3 Mechanical Properties

$$Y = \frac{\sigma}{\varepsilon} = \frac{F}{A}\frac{l}{\Delta l}. \tag{45}$$

$$\sigma = Y\varepsilon. \tag{46}$$

$$F = \frac{YA}{l}\Delta l, \tag{47}$$

$$G = \frac{\tau}{\alpha}. \tag{48}$$

$$K = -p\frac{V}{\Delta V}, \tag{49}$$

$$\frac{\Delta l_2}{l_2} = \frac{\Delta l_3}{l_3} = -\nu\frac{\Delta l_1}{l_1} = -\nu\varepsilon. \tag{50}$$

$$(l_1 + \Delta l_1)(l_2 + \Delta l_2)(l_3 + \Delta l_3). \tag{51}$$

$$l_1 l_2 l_3 + \Delta l_1 l_2 l_3 + l_1 \Delta l_2 l_3 + l_1 l_2 \Delta l_3. \tag{52}$$

$$\begin{aligned}\Delta l_1 l_2 l_3 + l_1 \Delta l_2 l_3 + l_1 l_2 \Delta l_3 &= \Delta l_1 l_2 l_3 + l_1\left(-\nu\frac{\Delta l_1}{l_1}l_2\right)l_3 + l_1 l_2\left(-\nu\frac{\Delta l_1}{l_1}l_3\right)\\ &= (1-2\nu)\Delta l_1 l_2 l_3.\end{aligned} \tag{53}$$

$$G = \frac{Y}{2(1+\nu)}. \tag{54}$$

$$\phi(x) = \phi(a) + \frac{\phi'(a)}{1!}(x-a) + \frac{\phi''(a)}{2!}(x-a)^2 + \frac{\phi'''}{3!}(x-a)^3 + \ldots \tag{55}$$

$$\alpha = \tan^{-1}\left(\frac{x}{a}\right) \approx \frac{x}{a}. \tag{56}$$

$$\tau = G\alpha \approx \frac{Gx}{a}, \tag{57}$$

$$\tau = C\sin\left(\frac{2\pi x}{b}\right), \tag{58}$$

$$C\frac{2\pi x}{b} = \frac{Gx}{a} \tag{59}$$

$$C = \tau_{\mathrm{y}} = \frac{Gb}{2\pi a}. \tag{60}$$

0.4 Thermal Properties of the Lattice

$$M\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\gamma x, \tag{61}$$

$$\omega = \sqrt{\frac{\gamma}{M}}. \tag{62}$$

$$E = \frac{1}{2}Mv^2 + \frac{1}{2}\gamma x^2. \tag{63}$$

$$\frac{1}{2}\gamma x_{\mathrm{max}}^2 = k_{\mathrm{B}}T, \tag{64}$$

$$x_{\mathrm{max}} = \left(\frac{2k_{\mathrm{B}}T}{\gamma}\right)^{1/2}. \tag{65}$$

$$M\frac{\mathrm{d}^2u_n}{\mathrm{d}t^2} = -\gamma(u_n - u_{n-1}) + \gamma(u_{n+1} - u_n), \tag{66}$$

$$M\frac{\mathrm{d}^2u_n}{\mathrm{d}t^2} = -\gamma[2u_n - u_{n-1} - u_{n+1}]. \tag{67}$$

$$u_n(t) = u\,\mathrm{e}^{\mathrm{i}(kan-\omega t)}, \tag{68}$$

$$\begin{aligned} -M\omega^2\mathrm{e}^{\mathrm{i}(kan-\omega t)} &= -\gamma\left[2 - \mathrm{e}^{-\mathrm{i}ka} - \mathrm{e}^{\mathrm{i}ka}\right]\mathrm{e}^{\mathrm{i}(kan-\omega t)} \\ &= -2\gamma(1-\cos ka)\mathrm{e}^{\mathrm{i}(kan-\omega t)}, \end{aligned} \tag{69}$$

$$\omega(k) = \sqrt{\frac{2\gamma(1-\cos ka)}{M}} = 2\sqrt{\frac{\gamma}{M}}\left|\sin\frac{ka}{2}\right|. \tag{70}$$

$$\omega(k) = \sqrt{\frac{\gamma}{M}}ak = vk, \tag{71}$$

$$u_{n+1}(t) = u\,\mathrm{e}^{\mathrm{i}(ka(n+1)-\omega t)} = \mathrm{e}^{\mathrm{i}ka}u_n(t). \tag{72}$$

$$\begin{aligned} M_1\frac{\mathrm{d}^2u_n}{\mathrm{d}t^2} &= -\gamma[2u_n - v_{n-1} - v_n], \\ M_2\frac{\mathrm{d}^2v_n}{\mathrm{d}t^2} &= -\gamma[2v_n - u_n - u_{n+1}], \end{aligned} \tag{73}$$

$$\begin{aligned} u_n(t) &= u\,\mathrm{e}^{\mathrm{i}(kbn-\omega t)}, \\ v_n(t) &= v\,\mathrm{e}^{\mathrm{i}(kbn-\omega t)}. \end{aligned} \tag{74}$$

$$\begin{aligned} -\omega^2M_1u &= \gamma v(1+\mathrm{e}^{-\mathrm{i}kb}) - 2\gamma u, \\ -\omega^2M_2v &= \gamma u(\mathrm{e}^{\mathrm{i}kb}+1) - 2\gamma v. \end{aligned} \tag{75}$$

$$\begin{vmatrix} 2\gamma - \omega^2M_1 & -\gamma(\mathrm{e}^{-\mathrm{i}kb}+1) \\ -\gamma(1+\mathrm{e}^{\mathrm{i}kb}) & 2\gamma - \omega^2M_2 \end{vmatrix} = 0. \tag{76}$$

$$\omega^2 = \gamma\left(\frac{1}{M_1}+\frac{1}{M_2}\right) \pm \gamma\left[\left(\frac{1}{M_1}+\frac{1}{M_2}\right)^2 - \frac{4}{M_1M_2}\sin^2\frac{kb}{2}\right]^{1/2}, \tag{77}$$

$$u_{N+n}(t) = u_n(t). \tag{78}$$

$$\mathrm{e}^{\mathrm{i}kan} = \mathrm{e}^{\mathrm{i}ka(N+n)}, \tag{79}$$

$$\mathrm{e}^{\mathrm{i}kNa} = 1, \tag{80}$$

$$k = \frac{2\pi}{aN}m, \tag{81}$$

$$E_l = \left(l + \frac{1}{2}\right)\hbar\omega \tag{82}$$

$$E_l(k) = \left(l + \frac{1}{2}\right)\hbar\omega(k). \tag{83}$$

$$\mathbf{k} = (k_x, k_y, k_z) = \frac{2\pi}{aN}(n_x, n_y, n_z) = \left(\frac{n_x 2\pi}{L}, \frac{n_y 2\pi}{L} \frac{n_z 2\pi}{L}\right), \tag{84}$$

$$\sigma = \frac{F}{a^2}. \tag{85}$$

$$F = \gamma\Delta a \tag{86}$$

$$\sigma = \frac{\gamma\Delta a}{a^2}. \tag{87}$$

$$Y = \frac{\sigma}{\varepsilon} = \frac{\gamma\Delta a}{a^2}\frac{a}{\Delta a} = \frac{\gamma}{a}. \tag{88}$$

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega_{\mathrm{E}}. \tag{89}$$

$$\langle E\rangle = 3N_{\mathrm{A}}\left(\langle n\rangle + \frac{1}{2}\right)\hbar\omega_{\mathrm{E}}. \tag{90}$$

$$\langle n\rangle = \frac{1}{\mathrm{e}^{\hbar\omega_{\mathrm{E}}/k_{\mathrm{B}}T} - 1}. \tag{91}$$

$$\langle E\rangle = 3N_{\mathrm{A}}\left(\frac{1}{\mathrm{e}^{\hbar\omega_{\mathrm{E}}/k_{\mathrm{B}}T} - 1} + \frac{1}{2}\right)\hbar\omega_{\mathrm{E}}. \tag{92}$$

$$C = \frac{\partial\langle E\rangle}{\partial T} = 3R\left(\frac{\hbar\omega_{\mathrm{E}}}{k_{\mathrm{B}}T}\right)^2 \frac{\mathrm{e}^{\hbar\omega_{\mathrm{E}}/k_{\mathrm{B}}T}}{(\mathrm{e}^{\hbar\omega_{\mathrm{E}}/k_{\mathrm{B}}T} - 1)^2}. \tag{93}$$

$$p_1 \propto \mathrm{e}^{-\hbar\omega_{\mathrm{E}}/k_{\mathrm{B}}T}, \tag{94}$$

$$\langle E\rangle = \frac{\hbar\omega}{\mathrm{e}^{\hbar\omega/k_{\mathrm{B}}T} - 1} \tag{95}$$

$$\langle E\rangle = 3\int_0^{\omega_{\mathrm{D}}} \frac{\hbar\omega}{\mathrm{e}^{\hbar\omega/k_{\mathrm{B}}T} - 1}\,\mathrm{d}\omega, \tag{96}$$

$$\langle E\rangle = 3\int_0^{\omega_{\mathrm{D}}} \frac{g(\omega)\hbar\omega}{\mathrm{e}^{\hbar\omega/k_{\mathrm{B}}T} - 1}\,\mathrm{d}\omega \tag{97}$$

$$N = \frac{4}{3}\pi n^3, \tag{98}$$

$$N = \frac{4}{3}\pi\left(\frac{Lk}{2\pi}\right)^3. \tag{99}$$

$$N(\omega) = \frac{4}{3}\pi\left(\frac{L\omega}{2\pi\nu}\right)^3 = \frac{V}{6\pi^2\nu^3}\omega^3, \tag{100}$$

$$g(\omega) = \frac{\mathrm{d}N}{\mathrm{d}\omega} = \frac{\omega^2 V}{2\pi^2\nu^3}. \tag{101}$$

$$3N = 3\int_0^{\omega_\mathrm{D}} g(\omega)\,\mathrm{d}\omega. \tag{102}$$

$$\omega_\mathrm{D}^3 = 6\pi^2\frac{N}{V}\nu^3. \tag{103}$$

$$\langle E\rangle = 3\int_0^{\omega_\mathrm{D}} \frac{\omega^2 V}{2\pi^2\nu^3}\frac{\hbar\omega}{\mathrm{e}^{\hbar\omega/k_\mathrm{B}T}-1}\,\mathrm{d}\omega = \frac{3V\hbar}{2\pi^2\nu^3}\int_0^{\omega_\mathrm{D}}\frac{\omega^3}{\mathrm{e}^{\hbar\omega/k_\mathrm{B}T}-1}\,\mathrm{d}\omega, \tag{104}$$

$$\langle E\rangle = \frac{3Vk_\mathrm{B}^4T^4}{2\pi^2\nu^3\hbar^3}\int_0^{x_\mathrm{D}}\frac{x^3}{\mathrm{e}^x-1}\,\mathrm{d}x = 9Nk_\mathrm{B}T\left(\frac{T}{\Theta_\mathrm{D}}\right)^3\int_0^{x_\mathrm{D}}\frac{x^3}{\mathrm{e}^x-1}\,\mathrm{d}x. \tag{105}$$

$$C = \frac{12\pi^4}{5}Nk_\mathrm{B}\left(\frac{T}{\Theta_\mathrm{D}}\right)^3. \tag{106}$$

$$\kappa = \kappa_\mathrm{p} + \kappa_\mathrm{e}. \tag{107}$$

$$\kappa = \frac{1}{A}\frac{\partial Q}{\partial t}\frac{\Delta x}{\Delta T}. \tag{108}$$

$$\kappa_\mathrm{p} = \frac{1}{3}c\lambda_\mathrm{p}v_\mathrm{p}, \tag{109}$$

$$\frac{\Delta l}{l} = \alpha\Delta T, \tag{110}$$

$$G = U + PV - TS. \tag{111}$$

$$T_\mathrm{m} = \frac{(0.05a)^2\gamma}{2k_\mathrm{B}} = \frac{(0.05a)^2\omega^2 M}{2k_\mathrm{B}}. \tag{112}$$

$$T_\mathrm{m} = \frac{(0.05a)^2\Theta_\mathrm{D}^2 k_\mathrm{B}M}{2\hbar^2}. \tag{113}$$

0.5 Electronic Properties of Metals: Classical Approach

$$\frac{1}{2}m_\mathrm{e}v_\mathrm{t}^2 = \frac{3}{2}k_\mathrm{B}T. \tag{114}$$

$$m_\mathrm{e}\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -e\boldsymbol{\mathcal{E}} \tag{115}$$

$$\mathbf{v}(t) = \frac{-e\boldsymbol{\mathcal{E}}t}{m_\mathrm{e}}, \tag{116}$$

$$\bar{\mathbf{v}} = \frac{-e\boldsymbol{\mathcal{E}}\tau}{m_\mathrm{e}}. \tag{117}$$

$$n|\bar{\mathbf{v}}|A, \tag{118}$$

$$-en|\bar{\mathbf{v}}|A. \tag{119}$$

$$\mathbf{j} = -en\overline{\mathbf{v}}, \tag{120}$$

$$\mathbf{j} = \frac{ne^2\tau}{m_\mathrm{e}}\boldsymbol{\mathcal{E}} = \sigma\boldsymbol{\mathcal{E}} = \frac{\boldsymbol{\mathcal{E}}}{\rho}, \tag{121}$$

$$\sigma = \frac{ne^2\tau}{m_\mathrm{e}}, \tag{122}$$

$$\rho = \frac{m_\mathrm{e}}{ne^2\tau}. \tag{123}$$

$$\mu = \frac{e\tau}{m_\mathrm{e}}, \tag{124}$$

$$\sigma = n\mu e, \qquad \rho = \frac{1}{n\mu e}. \tag{125}$$

$$\mathcal{E}_\mathrm{H} = R_\mathrm{H} j_x B_z, \tag{126}$$

$$|-e\mathcal{E}_\mathrm{H}| = |-eB_z v_x|. \tag{127}$$

$$R_\mathrm{H} = \frac{\mathcal{E}_\mathrm{H}}{j_x B_z} = \frac{\mathcal{E}_\mathrm{H}}{-env_x B_z} = \frac{v_x B_z}{-env_x B_z} = \frac{-1}{ne}. \tag{128}$$

$$R_\mathrm{H} = \frac{1}{pe}, \tag{129}$$

$$\boldsymbol{\mathcal{E}}(z,t) = \boldsymbol{\mathcal{E}}_0\,\mathrm{e}^{\mathrm{i}(kz-\omega t)}, \tag{130}$$

$$k = \frac{2\pi N}{\lambda_0}, \tag{131}$$

$$N = n + \mathrm{i}\kappa \tag{132}$$

$$N = \sqrt{\varepsilon} = \sqrt{\varepsilon_\mathrm{r} + \mathrm{i}\varepsilon_\mathrm{i}} \tag{133}$$

$$\boldsymbol{\mathcal{E}}(z,t) = \boldsymbol{\mathcal{E}}_0\,\mathrm{e}^{\mathrm{i}((2\pi N/\lambda_0)z-\omega t)} = \boldsymbol{\mathcal{E}}_0\,\mathrm{e}^{\mathrm{i}((\omega\sqrt{\varepsilon}/c)z-\omega t)}. \tag{134}$$

$$m_\mathrm{e}\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} = -e\mathcal{E}_0\,\mathrm{e}^{-\mathrm{i}\omega t}. \tag{135}$$

$$x(t) = A\,\mathrm{e}^{-\mathrm{i}\omega t}, \tag{136}$$

$$A = \frac{e\mathcal{E}_0}{m_\mathrm{e}\omega^2}. \tag{137}$$

$$P(t) = -nex(t) = -neA\mathrm{e}^{-\mathrm{i}\omega t} = -\frac{ne^2\mathcal{E}_0\,\mathrm{e}^{-\mathrm{i}\omega t}}{m_\mathrm{e}\omega^2}. \tag{138}$$

$$D = \varepsilon\varepsilon_0\mathcal{E} = \varepsilon_0\mathcal{E} + P, \tag{139}$$

$$\varepsilon = 1 + \frac{P(t)}{\varepsilon_0\mathcal{E}_0\,\mathrm{e}^{-\mathrm{i}\omega t}}. \tag{140}$$

$$\varepsilon = 1 - \frac{ne^2}{\varepsilon_0 m_\mathrm{e}\omega^2} = 1 - \frac{\omega_\mathrm{p}^2}{\omega^2}, \tag{141}$$

$$\omega_\mathrm{p}^2 = \frac{ne^2}{m_\mathrm{e}\varepsilon_0}. \tag{142}$$

$$\frac{\kappa}{\sigma} = LT, \tag{143}$$

$$\frac{\kappa}{\sigma} = \frac{3}{2}\frac{k_\mathrm{B}^2}{e^2}T = LT, \tag{144}$$

0.6 Electronic Properties of Solids: Quantum Mechanical Approach

$$-\frac{\hbar^2\nabla^2}{2m_e}\psi(\mathbf{r})+U(\mathbf{r})\psi(\mathbf{r})=E\psi(\mathbf{r}). \tag{145}$$

$$U(\mathbf{r})=U(\mathbf{r}+\mathbf{R}), \tag{146}$$

$$\psi(\mathbf{r})=\psi(x,y,z)=\psi(x+L,y,z)=\psi(x,y+L,z)=\psi(x,y,z+L). \tag{147}$$

$$\psi(\mathbf{r})=\frac{1}{\sqrt{V}}\,e^{i\mathbf{k}\cdot\mathbf{r}}. \tag{148}$$

$$\mathbf{k}=(k_x,k_y,k_z)=\left(\frac{n_x2\pi}{L},\frac{n_y2\pi}{L},\frac{n_z2\pi}{L}\right), \tag{149}$$

$$E(\mathbf{k})=\frac{\hbar^2k^2}{2m_e}=\frac{\hbar^2}{2m_e}(k_x^2+k_y^2+k_z^2). \tag{150}$$

$$\frac{\hbar^2}{2m_e}\left(\frac{2\pi}{L}\right)^2 \tag{151}$$

$$\frac{N}{2}=\frac{4}{3}\pi\left(\frac{Lk}{2\pi}\right)^3, \tag{152}$$

$$k_F=\left(\frac{3\pi^2N}{L^3}\right)^{1/3}=\left(3\pi^2\,n\right)^{1/3}. \tag{153}$$

$$E_F=\frac{\hbar^2k_F^2}{2m_e}=\frac{\hbar^2}{2m_e}\left(\frac{3\pi^2N}{L^3}\right)^{2/3}=\frac{\hbar^2}{2m_e}\left(3\pi^2n\right)^{2/3}. \tag{154}$$

$$N(E)=\frac{V}{3\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}E^{3/2}. \tag{155}$$

$$g(E)=\frac{dN}{dE}=\frac{V}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}E^{1/2}. \tag{156}$$

$$f(E,T)=\frac{1}{e^{(E-\mu)/k_BT}+1}, \tag{157}$$

$$N=\int_0^\infty g(E)f(E,T)\,dE. \tag{158}$$

$$\langle E\rangle=\frac{3}{2}k_BT\,g(E_F)k_BT \tag{159}$$

$$C=\frac{\partial\langle E\rangle}{\partial T}=3k_B^2T\,g(E_F). \tag{160}$$

$$C=\frac{\pi^2}{3}k_B^2T\,g(E_F) \tag{161}$$

$$\frac{\kappa}{\sigma}=\frac{\pi^2}{3}\frac{k_B^2}{e^2}T=LT. \tag{162}$$

$$\phi_0(r)=\frac{1}{4\pi\varepsilon_0}\frac{q}{r}, \tag{163}$$

$$\rho(r) = -\frac{e^2}{V} g(E_\mathrm{F})\phi(r). \tag{164}$$

$$\nabla^2\phi(\mathbf{r}) = \frac{\partial^2\phi(r)}{\partial r^2} + \frac{2}{r}\frac{\partial\phi(r)}{\partial r} = \frac{e^2}{V\varepsilon_0} g(E_\mathrm{F})\phi(r). \tag{165}$$

$$\phi(r) = c\frac{1}{r}\,\mathrm{e}^{-r/r_\mathrm{TF}}, \tag{166}$$

$$r_\mathrm{TF} = \sqrt{\frac{V\varepsilon_0}{e^2 g(E_\mathrm{F})}}. \tag{167}$$

$$\phi(r) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\,\mathrm{e}^{-r/r_\mathrm{TF}}, \tag{168}$$

$$\psi_\mathbf{k}(\mathbf{r}) = \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} u_\mathbf{k}(\mathbf{r}), \tag{169}$$

$$u_\mathbf{k}(\mathbf{r}) = u_\mathbf{k}(\mathbf{r}+\mathbf{R}), \tag{170}$$

$$\psi_\mathbf{k}(\mathbf{r}+\mathbf{R}) = \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{R}}\psi_\mathbf{k}(\mathbf{r}). \tag{171}$$

$$\psi(\mathbf{r}) = \sum_\mathbf{k} c_\mathbf{k}\,\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}. \tag{172}$$

$$U(\mathbf{r}) = \sum_\mathbf{G} U_\mathbf{G}\,\mathrm{e}^{\mathrm{i}\mathbf{G}\cdot\mathbf{r}}. \tag{173}$$

$$U_{-\mathbf{G}} = U^*_\mathbf{G}. \tag{174}$$

$$-\frac{\hbar^2\nabla^2}{2m_\mathrm{e}}\psi(\mathbf{r}) = \sum_\mathbf{k}\frac{\hbar^2 k^2}{2m_\mathrm{e}} c_\mathbf{k}\,\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}. \tag{175}$$

$$\begin{aligned} U(\mathbf{r})\psi(\mathbf{r}) &= \left(\sum_\mathbf{G} U_\mathbf{G}\,\mathrm{e}^{\mathrm{i}\mathbf{G}\cdot\mathbf{r}}\right)\left(\sum_\mathbf{k} c_\mathbf{k}\,\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\right) \\ &= \sum_\mathbf{k}\sum_\mathbf{G} U_\mathbf{G} c_\mathbf{k}\,\mathrm{e}^{\mathrm{i}(\mathbf{G}+\mathbf{k})\cdot\mathbf{r}} \\ &= \sum_{\mathbf{k}'}\sum_\mathbf{G} U_\mathbf{G} c_{\mathbf{k}'-\mathbf{G}}\,\mathrm{e}^{\mathrm{i}\mathbf{k}'\cdot\mathbf{r}}. \end{aligned} \tag{176}$$

$$\sum_\mathbf{k}\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\left\{\left(\frac{\hbar^2k^2}{2m_\mathrm{e}} - E\right)c_\mathbf{k} + \sum_\mathbf{G} U_\mathbf{G} c_{\mathbf{k}-\mathbf{G}}\right\} = 0. \tag{177}$$

$$\left(\frac{\hbar^2k^2}{2m_\mathrm{e}} - E\right)c_\mathbf{k} + \sum_\mathbf{G} U_\mathbf{G} c_{\mathbf{k}-\mathbf{G}} = 0. \tag{178}$$

$$\psi_\mathbf{k}(\mathbf{r}) = \sum_\mathbf{G} c_{\mathbf{k}-\mathbf{G}}\,\mathrm{e}^{\mathrm{i}(\mathbf{k}-\mathbf{G})\cdot\mathbf{r}}. \tag{179}$$

$$\psi_\mathbf{k}(\mathbf{r}) = \mathrm{e}^{\mathrm{i}\mathbf{k}\mathbf{r}}\left(\sum_\mathbf{G} c_{\mathbf{k}-\mathbf{G}}\,\mathrm{e}^{-\mathrm{i}\mathbf{G}\cdot\mathbf{r}}\right). \tag{180}$$

$$\psi_{\mathbf{k}+\mathbf{G}'}(\mathbf{r}) = \sum_\mathbf{G} c_{\mathbf{k}-\mathbf{G}+\mathbf{G}'}\,\mathrm{e}^{\mathrm{i}(\mathbf{k}-\mathbf{G}+\mathbf{G}')\cdot\mathbf{r}} = \sum_{\mathbf{G}''} c_{\mathbf{k}-\mathbf{G}''}\,\mathrm{e}^{\mathrm{i}(\mathbf{k}-\mathbf{G}'')\cdot\mathbf{r}} \tag{181}$$

$$\psi_{\mathbf{k}+\mathbf{G}'} = \psi_\mathbf{k}(\mathbf{r}), \tag{182}$$

$$E(\mathbf{k} + \mathbf{G}') = E(\mathbf{k}). \tag{183}$$

$$\left(\frac{\hbar^2 k^2}{2m_\mathrm{e}} - E \right) c_k + \sum_g U_g c_{k-g} = 0. \tag{184}$$

$$U(x) = \sum_g U_g \, \mathrm{e}^{\mathrm{i}gx}. \tag{185}$$

$$\left(\frac{\hbar^2 (k - g_1)^2}{2m_\mathrm{e}} - E \right) c_{k-g_1} + \sum_g U_g c_{k-g-g_1} = 0. \tag{186}$$

$$\begin{aligned} \left(\frac{\hbar^2 k^2}{2m_\mathrm{e}} - E \right) c_k + U c_{k+g_1} + U c_{k-g_1} &= 0, \\ \left(\frac{\hbar^2 (k - g_1)^2}{2m_\mathrm{e}} - E \right) c_{k-g_1} + U c_k + U c_{k-g_2} &= 0. \end{aligned} \tag{187}$$

$$\psi_k(x) = \sum_g c_{k-g} \, \mathrm{e}^{\mathrm{i}(k-g)x} \approx c_k \, \mathrm{e}^{\mathrm{i}kx} + c_{k-g_1} \, \mathrm{e}^{\mathrm{i}(k-g_1)x}. \tag{188}$$

$$\begin{aligned} \left(\frac{\hbar^2 k^2}{2m_\mathrm{e}} - E \right) c_k + U c_{k-g_1} &= 0, \\ U c_k + \left(\frac{\hbar^2 (k - g_1)^2}{2m_\mathrm{e}} - E \right) c_{k-g_1} &= 0, \end{aligned} \tag{189}$$

$$E_0 = \frac{\hbar^2}{2m_\mathrm{e}} \left(\pm \frac{\pi}{a} \right)^2. \tag{190}$$

$$\begin{vmatrix} E_0 - E & U \\ U & E_0 - E \end{vmatrix} = 0. \tag{191}$$

$$\psi(+) \propto \mathrm{e}^{\mathrm{i}(\pi/a)x} + \mathrm{e}^{-\mathrm{i}(\pi/a)x} = 2\cos\left(\frac{\pi}{a} x \right), \tag{192}$$

$$\psi(-) \propto \mathrm{e}^{\mathrm{i}(\pi/a)x} - \mathrm{e}^{-\mathrm{i}(\pi/a)x} = 2\mathrm{i}\sin\left(\frac{\pi}{a} x \right). \tag{193}$$

$$-\mathrm{i}\hbar \, \nabla \psi_\mathbf{k}(\mathbf{r}) = \hbar \mathbf{k} \, \psi_\mathbf{k}(\mathbf{r}) - \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} \, \mathrm{i} \, \hbar \, \nabla u_\mathbf{k}(\mathbf{r}). \tag{194}$$

$$v_\mathrm{g} = \frac{\mathrm{d}\omega(k)}{\mathrm{d}k} = \frac{1}{\hbar} \frac{\mathrm{d}E(k)}{\mathrm{d}k}. \tag{195}$$

$$H_\mathrm{at} = -\frac{\hbar^2 \nabla^2}{2m_\mathrm{e}} + V_\mathrm{at}(\mathbf{r}), \tag{196}$$

$$\begin{aligned} H_\mathrm{sol} &= -\frac{\hbar^2 \nabla^2}{2m_\mathrm{e}} + \sum_\mathbf{R} V_\mathrm{at}(\mathbf{r} - \mathbf{R}) \\ &= -\frac{\hbar^2 \nabla^2}{2m_\mathrm{e}} + V_\mathrm{at}(\mathbf{r}) + \sum_{\mathbf{R} \neq 0} V_\mathrm{at}(\mathbf{r} - \mathbf{R}). \end{aligned} \tag{197}$$

$$H_{\mathrm{sol}} = -\frac{\hbar^2\nabla^2}{2m_{\mathrm{e}}} + V_{\mathrm{at}}(\mathbf{r}) + v(\mathbf{r}) = H_{\mathrm{at}} + v(\mathbf{r}), \tag{198}$$

$$v(\mathbf{r}) = \sum_{\mathbf{R}\neq 0} V_{\mathrm{at}}(\mathbf{r}-\mathbf{R}). \tag{199}$$

$$\int \phi_n^*(\mathbf{r}) H_{\mathrm{sol}} \phi_n(\mathbf{r})\,\mathrm{d}\mathbf{r} = E_n + \int \phi_n^*(\mathbf{r}) v(\mathbf{r}) \phi_n(\mathbf{r})\,\mathrm{d}\mathbf{r} = E_n - \beta, \tag{200}$$

$$\psi_{\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{N}} \sum_{\mathbf{R}} c_{\mathbf{k},\mathbf{R}} \phi_n(\mathbf{r}-\mathbf{R}). \tag{201}$$

$$\psi_{\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{N}} \sum_{\mathbf{R}} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{R}} \phi_n(\mathbf{r}-\mathbf{R}), \tag{202}$$

$$\begin{aligned} \psi_{\mathbf{k}}(\mathbf{r}+\mathbf{R}') &= \frac{1}{\sqrt{N}} \sum_{\mathbf{R}} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{R}} \phi_n(\mathbf{r}-\mathbf{R}+\mathbf{R}') \\ &= \frac{1}{\sqrt{N}} \mathrm{e}^{\mathrm{i}\mathbf{k}\mathbf{R}'} \sum_{\mathbf{R}} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{R}-\mathbf{R}')} \phi_n\big(\mathbf{r}-(\mathbf{R}-\mathbf{R}')\big) \\ &= \frac{1}{\sqrt{N}} \mathrm{e}^{\mathrm{i}\mathbf{k}\mathbf{R}'} \sum_{\mathbf{R}''} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{R}''} \phi_n(\mathbf{r}-\mathbf{R}'') = \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{R}'} \psi_{\mathbf{k}}(\mathbf{r}), \end{aligned} \tag{203}$$

$$\begin{aligned} E(\mathbf{k}) &= \int \psi_{\mathbf{k}}^*(\mathbf{r}) H_{\mathrm{sol}} \psi_{\mathbf{k}}(\mathbf{r})\,\mathrm{d}\mathbf{r} \\ &= \frac{1}{N} \sum_{\mathbf{R}} \sum_{\mathbf{R}'} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{R}-\mathbf{R}')} \int \phi_n^*(\mathbf{r}-\mathbf{R}') H_{\mathrm{sol}} \phi_n(\mathbf{r}-\mathbf{R})\,\mathrm{d}\mathbf{r}, \end{aligned} \tag{204}$$

$$E(\mathbf{k}) = \sum_{\mathbf{R}} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{R}} \int \phi_n^*(\mathbf{r}) H_{\mathrm{sol}} \phi_n(\mathbf{r}-\mathbf{R})\,\mathrm{d}\mathbf{r}. \tag{205}$$

$$E(\mathbf{k}) = E_n - \beta + \sum_{\mathbf{R}\neq 0} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{R}} \int \phi_n^*(\mathbf{r}) H_{\mathrm{sol}} \phi_n(\mathbf{r}-\mathbf{R})\,\mathrm{d}\mathbf{r}. \tag{206}$$

$$\begin{aligned} \int \phi_n^*(\mathbf{r}) H_{\mathrm{sol}} \phi_n(\mathbf{r}-\mathbf{R})\,\mathrm{d}\mathbf{r} = E_n &\int \phi_n^*(\mathbf{r}) \phi_n(\mathbf{r}-\mathbf{R})\,\mathrm{d}\mathbf{r} \\ &+ \int \phi_n^*(\mathbf{r}) v(\mathbf{r}) \phi_n(\mathbf{r}-\mathbf{R})\,\mathrm{d}\mathbf{r}. \end{aligned} \tag{207}$$

$$\gamma(\mathbf{R}) = -\int \phi_n^*(\mathbf{r}) v(\mathbf{r}) \phi_n(\mathbf{r}-\mathbf{R})\,\mathrm{d}\mathbf{r}, \tag{208}$$

$$E(\mathbf{k}) = E_n - \beta - \sum_{\mathbf{R}\neq 0} \gamma(\mathbf{R})\,\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{R}}. \tag{209}$$

$$E_{\mathrm{s}}(k) = E_{\mathrm{s}} - \beta_{\mathrm{s}} - \gamma_{\mathrm{s}}(\mathrm{e}^{\mathrm{i}ka} + \mathrm{e}^{-\mathrm{i}ka}) = E_{\mathrm{s}} - \beta_{\mathrm{s}} - 2\gamma_{\mathrm{s}} \cos ka, \tag{210}$$

$$\mathrm{d}E = -e\mathcal{E} v_{\mathrm{g}}\,\mathrm{d}t. \tag{211}$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\mathrm{d}E}{\mathrm{d}k}\frac{\mathrm{d}k}{\mathrm{d}t}, \tag{212}$$

$$\hbar\frac{\mathrm{d}k}{\mathrm{d}t} = -e\mathcal{E}. \tag{213}$$

$$a = \frac{\mathrm{d}v_{\mathrm{g}}}{\mathrm{d}t} = \frac{1}{\hbar}\frac{\mathrm{d}}{\mathrm{d}t}\frac{\mathrm{d}E(k)}{\mathrm{d}k} = \frac{1}{\hbar}\frac{\mathrm{d}^2E(k)}{\mathrm{d}k^2}\frac{\mathrm{d}k}{\mathrm{d}t}. \tag{214}$$

$$a = -\frac{1}{\hbar^2}\frac{\mathrm{d}^2E(k)}{\mathrm{d}k^2}e\mathcal{E}. \tag{215}$$

$$m^* = \hbar^2\left(\frac{\mathrm{d}^2E(k)}{\mathrm{d}k^2}\right)^{-1}. \tag{216}$$

0.7 Semiconductors

$$n = \frac{1}{V}\int_{E_{\mathrm{c}}}^{\infty} g_{\mathrm{c}}(E)f(E,T)\,\mathrm{d}E, \tag{217}$$

$$p = \frac{1}{V}\int_{-\infty}^{E_{\mathrm{v}}} g_{\mathrm{v}}(E)[1-f(E,T)]\,\mathrm{d}E, \tag{218}$$

$$E = E_{\mathrm{g}} + \frac{\hbar^2k^2}{2m_{\mathrm{e}}^*}. \tag{219}$$

$$g_{\mathrm{c}}(E) = \frac{V}{2\pi^2}\left(\frac{2m_{\mathrm{e}}^*}{\hbar^2}\right)^{3/2}(E-E_{\mathrm{g}})^{1/2}. \tag{220}$$

$$E = -\frac{\hbar^2k^2}{2m_{\mathrm{h}}^*} \tag{221}$$

$$g_{\mathrm{v}}(E) = \frac{V}{2\pi^2}\left(\frac{2m_{\mathrm{h}}^*}{\hbar^2}\right)^{3/2}(-E)^{1/2}. \tag{222}$$

$$f(E,T) = \frac{1}{\mathrm{e}^{(E-\mu)/k_{\mathrm{B}}T}+1} \approx \mathrm{e}^{-(E-\mu)/k_{\mathrm{B}}T}. \tag{223}$$

$$1-f(E,T) = 1-\frac{1}{\mathrm{e}^{(E-\mu)/k_{\mathrm{B}}T}+1} \approx \mathrm{e}^{(E-\mu)/k_{\mathrm{B}}T} \tag{224}$$

$$\begin{aligned} n &= \frac{1}{V}\int_{E_{\mathrm{g}}}^{\infty}\frac{V}{2\pi^2}\left(\frac{2m_{\mathrm{e}}^*}{\hbar^2}\right)^{3/2}(E-E_{\mathrm{g}})^{1/2}\,\mathrm{e}^{-(E-\mu)/k_{\mathrm{B}}T}\,\mathrm{d}E \\ &= \frac{(2m_{\mathrm{e}}^*)^{3/2}}{2\pi^2\hbar^3}\mathrm{e}^{\mu/k_{\mathrm{B}}T}\int_{E_{\mathrm{g}}}^{\infty}(E-E_{\mathrm{g}})^{1/2}\,\mathrm{e}^{-E/k_{\mathrm{B}}T}\,\mathrm{d}E. \end{aligned} \tag{225}$$

$$n = \frac{(2m_{\mathrm{e}}^*)^{3/2}}{2\pi^2\hbar^3}(k_{\mathrm{B}}T)^{3/2}\,\mathrm{e}^{-(E_{\mathrm{g}}-\mu)/k_{\mathrm{B}}T}\int_0^{\infty}X_{\mathrm{g}}^{1/2}\,\mathrm{e}^{-X_{\mathrm{g}}}\,\mathrm{d}X_{\mathrm{g}}. \tag{226}$$

$$n = \frac{1}{\sqrt{2}}\left(\frac{m_{\mathrm{e}}^*k_{\mathrm{B}}T}{\pi\hbar^2}\right)^{3/2}\mathrm{e}^{-(E_{\mathrm{g}}-\mu)/k_{\mathrm{B}}T} = N_{\mathrm{eff}}^{\mathrm{C}}\,\mathrm{e}^{-(E_{\mathrm{g}}-\mu)/k_{\mathrm{B}}T}, \tag{227}$$

$$p = \frac{1}{\sqrt{2}}\left(\frac{m_{\mathrm{h}}^*k_{\mathrm{B}}T}{\pi\hbar^2}\right)^{3/2}\mathrm{e}^{-\mu/k_{\mathrm{B}}T} = N_{\mathrm{eff}}^{V}\,\mathrm{e}^{-\mu/k_{\mathrm{B}}T}. \tag{228}$$

$$np = 4\left(\frac{k_{\mathrm{B}}T}{2\pi\hbar^2}\right)^3(m_{\mathrm{e}}^*m_{\mathrm{h}}^*)^{3/2}\,\mathrm{e}^{-E_{\mathrm{g}}/k_{\mathrm{B}}T}, \tag{229}$$

$$n_{\mathrm{i}} = p_{\mathrm{i}} = \sqrt{np} = 2\left(\frac{k_{\mathrm{B}}T}{2\pi\hbar^2}\right)^{3/2} (m_{\mathrm{e}}^* m_{\mathrm{h}}^*)^{3/4}\, \mathrm{e}^{-E_{\mathrm{g}}/2k_{\mathrm{B}}T}. \tag{230}$$

$$\mu = \frac{E_{\mathrm{g}}}{2} + \frac{3}{4}k_{\mathrm{B}}T \ln\left(\frac{m_{\mathrm{h}}^*}{m_{\mathrm{e}}^*}\right). \tag{231}$$

$$\omega_c = \frac{Be}{m_{\mathrm{e}}^*}. \tag{232}$$

$$E_n = -\frac{m_{\mathrm{e}}e^4}{8\varepsilon_0^2 h^2}\frac{1}{n^2}. \tag{233}$$

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_{\mathrm{e}}e^2}. \tag{234}$$

$$\sigma = e(n\mu_{\mathrm{e}} + p\mu_{\mathrm{h}}), \tag{235}$$

$$\frac{\mathrm{d}^2U}{\mathrm{d}x^2} = -\frac{\rho}{\varepsilon\varepsilon_0}, \tag{236}$$

$$\left.\frac{\mathrm{d}U}{\mathrm{d}x}\right|_{x=-d_{\mathrm{p}},d_{\mathrm{n}}} = 0. \tag{237}$$

$$\Delta U = \frac{e}{2\varepsilon\varepsilon_0}\left(N_{\mathrm{d}}d_{\mathrm{n}}^2 + N_{\mathrm{a}}d_{\mathrm{p}}^2\right). \tag{238}$$

$$d_{\mathrm{p}} = \left(\frac{\Delta U 2\varepsilon\varepsilon_0}{eN_{\mathrm{a}}}\frac{N_{\mathrm{d}}}{N_{\mathrm{a}}+N_{\mathrm{d}}}\right)^{1/2}, \qquad d_{\mathrm{n}} = \left(\frac{\Delta U 2\varepsilon\varepsilon_0}{eN_{\mathrm{d}}}\frac{N_{\mathrm{a}}}{N_{\mathrm{a}}+N_{\mathrm{d}}}\right)^{1/2}. \tag{239}$$

$$n_{\mathrm{p}} = N_{\mathrm{eff}}^{\mathrm{c}}\,\mathrm{e}^{(\mu-E_{\mathrm{g}}-e\Delta U)/k_{\mathrm{B}}T}, \qquad p_{\mathrm{p}} = N_{\mathrm{eff}}^{\mathrm{v}}\,\mathrm{e}^{(e\Delta U-\mu)/k_{\mathrm{B}}T}, \tag{240}$$

$$n_{\mathrm{n}} = N_{\mathrm{eff}}^{\mathrm{c}}\,\mathrm{e}^{(\mu-E_{\mathrm{g}})/k_{\mathrm{B}}T}, \qquad p_{\mathrm{n}} = N_{\mathrm{eff}}^{\mathrm{v}}\,\mathrm{e}^{-\mu/k_{\mathrm{B}}T}. \tag{241}$$

$$|I_{\mathrm{diffusion}}| = |I_{\mathrm{drift}}| = |I_0| = C\,\mathrm{e}^{(\mu-E_{\mathrm{g}}-e\Delta U)/k_{\mathrm{B}}T}, \tag{242}$$

$$|I_{\mathrm{diffusion}}| = C\,\mathrm{e}^{[(\mu+eV)-E_{\mathrm{g}}-e\Delta U]/k_{\mathrm{B}}T}. \tag{243}$$

$$I = I_{\mathrm{diffusion}} - I_{\mathrm{drift}} = I_0\left(\mathrm{e}^{eV/k_{\mathrm{B}}T} - 1\right). \tag{244}$$

$$|I_{\mathrm{diffusion}}| = C\,\mathrm{e}^{[(\mu-eV)-E_{\mathrm{g}}-e\Delta U]/k_{\mathrm{B}}T}, \tag{245}$$

$$I = I_{\mathrm{diffusion}} - I_{\mathrm{drift}} = I_0\left(\mathrm{e}^{-eV/k_{\mathrm{B}}T} - 1\right). \tag{246}$$

0.8 Magnetism

$$\oint \mathbf{B}\,\mathrm{d}\mathbf{a} = 0, \qquad \mathrm{div}\,\mathbf{B} = 0 \tag{247}$$

$$\mathbf{B} = \mu_0\mathbf{H}, \tag{248}$$

$$\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M}) = \mathbf{B}_0 + \mu_0\mathbf{M}, \tag{249}$$

$$\mathbf{M} = \boldsymbol{\mu}\frac{N}{V}. \tag{250}$$

$$\mu_0\mathbf{M} = \chi_{\mathrm{m}}\mathbf{B}_0, \tag{251}$$

$$U = -V\int_0^{B_0} M\,\mathrm{d}B_0' = -V\int_0^{B_0}\frac{\chi_\mathrm{m}}{\mu_0}B_0'\,\mathrm{d}B_0' = -V\frac{\chi_\mathrm{m}}{2\mu_0}B_0^2, \tag{252}$$

$$\mathbf{B} = \mathrm{curl}\,\mathbf{A}. \tag{253}$$

$$\mathbf{p} \to \mathbf{p} - q\mathbf{A}. \tag{254}$$

$$\mathbf{A} = -\frac{1}{2}\mathbf{r}\times\mathbf{B}_0, \tag{255}$$

$$H_\mathrm{kin} \to H_\mathrm{kin}',$$

$$\frac{\mathbf{p}^2}{2m_\mathrm{e}} \to \frac{1}{2m_\mathrm{e}}(\mathbf{p}+e\mathbf{A})^2 = \frac{1}{2m_\mathrm{e}}\left(\mathbf{p} - e\frac{\mathbf{r}\times\mathbf{B}_0}{2}\right)^2. \tag{256}$$

$$H_\mathrm{kin}' = \frac{1}{2m_\mathrm{e}}\left(\mathbf{p}^2 + e\mathbf{B}_0\cdot(\mathbf{r}\times\mathbf{p}) + \frac{e^2}{4}(\mathbf{r}\times\mathbf{B}_0)^2\right), \tag{257}$$

$$H_\mathrm{kin}' = H_\mathrm{kin} + H' = \frac{\mathbf{p}^2}{2m_\mathrm{e}} + \frac{e}{2m_\mathrm{e}}B_0(\mathbf{r}\times\mathbf{p})_z + \frac{e^2}{8m_\mathrm{e}}B_0^2(x^2+y^2). \tag{258}$$

$$E' = \frac{e}{2m_\mathrm{e}}B_0\langle\psi|(\mathbf{r}\times\mathbf{p})_z|\psi\rangle + \frac{e^2}{8m_\mathrm{e}}B_0^2\langle\psi|(x^2+y^2)|\psi\rangle. \tag{259}$$

$$g_\mathrm{e}m_\mathrm{s}\frac{e\hbar}{2m_\mathrm{e}}B_0 = g_\mathrm{e}m_\mathrm{s}\mu_\mathrm{B}B_0, \tag{260}$$

$$\boldsymbol{\mu} = -\frac{e\hbar}{2m_\mathrm{e}}\mathbf{L} = -\mu_\mathrm{B}\mathbf{L}. \tag{261}$$

$$\mu_l = -\frac{em_l\hbar}{2m_\mathrm{e}} = -m_l\mu_\mathrm{B}. \tag{262}$$

$$\boldsymbol{\mu} = -g_\mathrm{e}\mu_\mathrm{B}\mathbf{S}, \tag{263}$$

$$\mu_\mathrm{s} = -g_\mathrm{e}m_\mathrm{s}\mu_\mathrm{B}, \tag{264}$$

$$\mu_J = -gm_J\mu_\mathrm{B}, \tag{265}$$

$$L = \sum m_l \quad \text{and} \quad S = \sum m_\mathrm{s}, \tag{266}$$

$$g_J = \frac{3J(J+1)+S(S+1)-L(L+1)}{2J(J+1)}. \tag{267}$$

$$\mu = -\frac{\partial E'}{\partial B_0} = -\frac{e^2}{4m_\mathrm{e}}B_0\langle\psi|(x^2+y^2)|\psi\rangle. \tag{268}$$

$$\mu = -\frac{Ze^2}{6m_\mathrm{e}}r_\mathrm{a}^2B_0. \tag{269}$$

$$\chi_\mathrm{m} = \mu_0\frac{M}{B_0} = -\frac{\mu_0 ZNe^2}{6Vm_\mathrm{e}}r_\mathrm{a}^2. \tag{270}$$

$$\chi_\mathrm{m} = -\frac{1}{3V}\mu_\mathrm{B}^2\,\mu_0\,g(E_\mathrm{F})\left(\frac{m_\mathrm{e}}{m^*}\right)^2. \tag{271}$$

$$\bar{\mu} = \frac{1}{Z}\sum_{m_J=-J}^{J} g_J\mu_\mathrm{B}m_J\,\mathrm{e}^{g_J\mu_\mathrm{B}m_JB_0/k_\mathrm{B}T}, \tag{272}$$

$$Z = \sum_{m_J=-J}^{J} \mathrm{e}^{-g_J \mu_\mathrm{B} m_J B_0 / k_\mathrm{B} T}. \tag{273}$$

$$\chi_\mathrm{m} = \frac{C}{T}, \tag{274}$$

$$C = \frac{\mu_0 N g_J^2 \mu_\mathrm{B}^2 J(J+1)}{3 V k_\mathrm{B}}. \tag{275}$$

$$N_{\downarrow\downarrow B_0} - N_{\downarrow\uparrow B_0} = g(E_\mathrm{F}) \mu_\mathrm{B} B_0, \tag{276}$$

$$M = \frac{1}{V}(N_{\downarrow\downarrow B_0} - N_{\downarrow\uparrow B_0}) \mu_\mathrm{B} = \frac{1}{V} g(E_\mathrm{F}) \mu_\mathrm{B}^2 B_0, \tag{277}$$

$$\chi_\mathrm{m} = \frac{1}{V} \mu_0 \, \mu_\mathrm{B}^2 \, g(E_\mathrm{F}), \tag{278}$$

$$E_{\uparrow\uparrow} - E_{\uparrow\downarrow} = -2X. \tag{279}$$

$$H = -2X \mathbf{S}_1 \cdot \mathbf{S}_2. \tag{280}$$

$$H = -\sum_i \sum_{j \neq i} X_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + g_\mathrm{e} \mu_\mathrm{B} \mathbf{B}_0 \cdot \sum_i \mathbf{S}_i, \tag{281}$$

$$H = -X \sum_i \sum_{\mathrm{n.n.}} \mathbf{S}_i \cdot \mathbf{S}_{\mathrm{n.n.}} + g_\mathrm{e} \mu_\mathrm{B} \mathbf{B}_0 \cdot \sum_i \mathbf{S}_i, \tag{282}$$

$$H = \sum_i \mathbf{S}_i \cdot \left(-\sum_{\mathrm{n.n.}} X \langle \mathbf{S} \rangle + g_\mathrm{e} \mu_\mathrm{B} \mathbf{B}_0 \right) = \sum_i \mathbf{S}_i \cdot \left(-n_{\mathrm{n.n.}} X \langle \mathbf{S} \rangle + g_\mathrm{e} \mu_\mathrm{B} \mathbf{B}_0 \right), \tag{283}$$

$$\mathbf{M} = -g_\mathrm{e} \mu_\mathrm{B} \langle \mathbf{S} \rangle \frac{N}{V}, \tag{284}$$

$$H = \sum_i \mathbf{S}_i \cdot \left(\frac{n_{\mathrm{n.n.}} X V}{g_\mathrm{e} \mu_\mathrm{B} N} \mathbf{M} + g_\mathrm{e} \mu_\mathrm{B} \mathbf{B}_0 \right) = g_\mathrm{e} \mu_\mathrm{B} \sum_i \mathbf{S}_i \cdot (\mathbf{B}_\mathrm{W} + \mathbf{B}_0) \tag{285}$$

$$\mathbf{B}_\mathrm{W} = \mathbf{M} \frac{n_{\mathrm{n.n.}} X V}{g_\mathrm{e}^2 \mu_\mathrm{B}^2 N}. \tag{286}$$

$$M(T) = \frac{\mu_\mathrm{B} N}{V} \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{\mathrm{e}^{-x} + \mathrm{e}^x} = M(0) \tanh(x), \tag{287}$$

$$\frac{M(T)}{M(0)} = \tanh\left(\frac{M(T)}{M(0)} \frac{\Theta_\mathrm{C}}{T} \right), \tag{288}$$

$$\Theta_\mathrm{C} = \frac{n_{\mathrm{n.n.}} X}{g_\mathrm{e}^2 k_\mathrm{B}} \tag{289}$$

$$\chi_\mathrm{m} = \frac{C}{T - \Theta_\mathrm{C}}. \tag{290}$$

0.9 Dielectrics

$$\mathbf{P} = \chi_e \varepsilon_0 \boldsymbol{\mathcal{E}}, \tag{291}$$

$$\mathbf{P} = \frac{N}{V}\mathbf{p} = \frac{N}{V} q \boldsymbol{\delta}. \tag{292}$$

$$\mathbf{p} = \alpha \boldsymbol{\mathcal{E}}, \tag{293}$$

$$\mathbf{P} = (\varepsilon - 1)\varepsilon_0 \boldsymbol{\mathcal{E}} = \frac{N}{V}\mathbf{p} = \frac{N}{V}\alpha \boldsymbol{\mathcal{E}}, \tag{294}$$

$$\alpha = \frac{(\varepsilon - 1)\varepsilon_0 V}{N}. \tag{295}$$

$$\boldsymbol{\mathcal{E}}_{\text{loc}} = \frac{1}{3}(\varepsilon + 2)\boldsymbol{\mathcal{E}}. \tag{296}$$

$$\mathbf{P} = \frac{N}{V}\alpha \boldsymbol{\mathcal{E}}_{\text{loc}} = \frac{N\alpha}{3V}(\varepsilon + 2)\boldsymbol{\mathcal{E}}. \tag{297}$$

$$\alpha = \frac{\varepsilon - 1}{\varepsilon + 2}\frac{3\varepsilon_0 V}{N}. \tag{298}$$

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + \eta \frac{\mathrm{d}x}{\mathrm{d}t} + \omega_0^2 x = \frac{e\mathcal{E}_0}{M}\,\mathrm{e}^{-\mathrm{i}\omega t}. \tag{299}$$

$$x(t) = A\,\mathrm{e}^{-\mathrm{i}\omega t}, \tag{300}$$

$$A = \frac{e\mathcal{E}_0}{M}\frac{1}{\omega_0^2 - \omega^2 - \mathrm{i}\eta\omega}. \tag{301}$$

$$A = \frac{e\mathcal{E}_0}{M}\left(\frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \eta^2\omega^2} + \frac{\mathrm{i}\eta\omega}{(\omega_0^2 - \omega^2)^2 + \eta^2\omega^2}\right). \tag{302}$$

$$P(t) = P_{\mathrm{i}}(t) + P_{\mathrm{e}}(t) = \frac{N}{V}eA\,\mathrm{e}^{-\mathrm{i}\omega t} + \frac{N}{V}\alpha\mathcal{E}_0\,\mathrm{e}^{-\mathrm{i}\omega t}. \tag{303}$$

$$\varepsilon = \frac{P(t)}{\varepsilon_0\mathcal{E}_0\,\mathrm{e}^{-\mathrm{i}\omega t}} + 1 = \frac{NeA}{V\varepsilon_0\mathcal{E}_0} + \frac{N\alpha}{V\varepsilon_0} + 1. \tag{304}$$

$$\varepsilon_{\text{opt}} = \frac{N\alpha}{V\varepsilon_0} + 1, \tag{305}$$

$$\varepsilon(\omega) = \frac{NeA}{V\varepsilon_0\mathcal{E}_0} + \varepsilon_{\text{opt}}. \tag{306}$$

$$\varepsilon_{\mathrm{r}}(\omega) = \frac{Ne^2}{V\varepsilon_0 M}\frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \eta^2\omega^2} + \varepsilon_{\text{opt}} \tag{307}$$

$$\varepsilon_{\mathrm{i}}(\omega) = \frac{Ne^2}{V\varepsilon_0 M}\frac{\eta\omega}{(\omega_0^2 - \omega^2)^2 + \eta^2\omega^2}. \tag{308}$$

$$p(t) = j(t)\mathcal{E}(t), \tag{309}$$

$$j(t) = -\frac{\partial D}{\partial t} = -\frac{\partial}{\partial t}\varepsilon\varepsilon_0\mathcal{E}(t) = \varepsilon_0\mathcal{E}(t)\left(\mathrm{i}\omega\varepsilon_{\mathrm{r}} - \omega\varepsilon_{\mathrm{i}}\right). \tag{310}$$

$$\overline{p} = \frac{1}{T}\int_0^T \mathcal{E}(t)j(t)\,\mathrm{d}t, \tag{311}$$

$$\frac{1}{2}\varepsilon_0\varepsilon_{\mathrm{i}}\omega\mathcal{E}_0^2. \tag{312}$$

$$\varepsilon_{\mathrm{i}}(h\nu) \propto \sum_{\mathbf{k}} M^2\delta\left[E_{\mathrm{c}}(\mathbf{k}) - E_{\mathrm{v}}(\mathbf{k}) - h\nu\right], \tag{313}$$

0.10 Superconductivity

$$B_c(T) = B_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]. \tag{314}$$

$$\oint \boldsymbol{\mathcal{E}}\,\mathrm{d}\mathbf{l} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}, \tag{315}$$

$$\frac{\partial \mathbf{j}}{\partial t} = \frac{n_s q^2}{m_s}\boldsymbol{\mathcal{E}}, \tag{316}$$

$$\frac{\partial}{\partial t}\left(\frac{m_s}{n_s q^2}\,\mathrm{curl}\,\mathbf{j} + \mathbf{B}\right) = 0. \tag{317}$$

$$\frac{\partial}{\partial t}\left(\int \frac{m_s}{n_s q^2}\,\mathrm{curl}\,\mathbf{j}\,\mathrm{d}\mathbf{A} + \int \mathbf{B}\,\mathrm{d}\mathbf{A}\right) = \frac{\partial}{\partial t}\left(\oint \frac{m_s}{n_s q^2}\mathbf{j}\,\mathrm{d}l + \int \mathbf{B}\,\mathrm{d}\mathbf{A}\right) = 0. \tag{318}$$

$$\frac{m_s}{n_s q^2}\,\mathrm{curl}\,\mathbf{j} + \mathbf{B} = 0. \tag{319}$$

$$\mathrm{curl}\,\mathbf{B} = \mu_0\,\mathbf{j}. \tag{320}$$

$$\mathrm{curl}\,\mathrm{curl}\,\mathbf{B} = \mu_0\,\mathrm{curl}\,\mathbf{j} = -\frac{\mu_0 n_s q^2}{m_s}\mathbf{B}. \tag{321}$$

$$\Delta\mathbf{B} = \frac{\mu_0 n_s q^2}{m_s}\mathbf{B}. \tag{322}$$

$$\Delta\mathbf{j} = \frac{\mu_0 n_s q^2}{m_s}\mathbf{j}. \tag{323}$$

$$\lambda_L = \sqrt{m_s/\mu_0 n_s q^2}. \tag{324}$$

$$\Psi(\mathbf{r}) = \Psi_0(\mathbf{r})\,\mathrm{e}^{\mathrm{i}\phi(\mathbf{r})}, \tag{325}$$

$$T_c = 1.13\Theta_D \exp\frac{-1}{g(E_F)V}, \tag{326}$$

$$\oint \frac{\mathbf{p}}{h}\,\mathrm{d}\mathbf{r} = n, \tag{327}$$

$$\oint \mathbf{p} - q\mathbf{A}\,\mathrm{d}\mathbf{r} = nh. \tag{328}$$

$$\frac{m_s}{n_s q}\oint \mathbf{j}\,\mathrm{d}\mathbf{r} - q\oint \mathbf{A}\,\mathrm{d}\mathbf{r} = nh. \tag{329}$$

$$\oint \mathbf{A}\,\mathrm{d}\mathbf{r} = \int \mathrm{curl}\,\mathbf{A}\,\mathrm{d}\mathbf{a} = \int \mathbf{B}\,\mathrm{d}\mathbf{a} = \Phi_B, \tag{330}$$

$$\frac{m_s}{n_s q^2}\oint \mathbf{j}\,\mathrm{d}\mathbf{r} - \Phi_B = n\frac{h}{q}. \tag{331}$$

$$\Phi_B = n\frac{h}{q}. \tag{332}$$

0.11 Finite Solids and Nanostructures

$$\Psi(\mathbf{r}) = \Psi(z)\,\Psi(x,y). \tag{333}$$

$$E_{xy} = \frac{\hbar^2\,k_{xy}^2}{2m_\mathrm{e}}, \tag{334}$$

$$\Psi(z) = A\,\mathrm{e}^{\mathrm{i}k_z z} + B\,\mathrm{e}^{-\mathrm{i}k_z z}, \tag{335}$$

$$k_z d = n\pi, \qquad n = 1,2,3,\ldots \tag{336}$$

$$E_z = \frac{\hbar^2 k_z^2}{2m_\mathrm{e}}, \tag{337}$$

$$2k_z d + \Phi_\mathrm{i} + \Phi_\mathrm{v} = 2\pi\,n, \qquad n = 1,2,3,\ldots \tag{338}$$

$$E_\mathrm{min} = E_\mathrm{g} + \frac{\hbar^2\pi^2}{2\mu r^2} - \frac{1.8e^2}{4\pi\,\varepsilon_0\varepsilon r} \tag{339}$$

$$\psi_\mathbf{k}(\mathbf{r}) = \mathrm{e}^{\mathrm{i}\mathbf{k}\mathbf{r}}u_\mathbf{k}(\mathbf{r}). \tag{340}$$

$$\psi_\mathbf{k}(\mathbf{r}) = \mathrm{e}^{\Im(k_z)z}\,\mathrm{e}^{\mathrm{i}\mathbf{k}'\cdot\mathbf{r}}u_\mathbf{k}(\mathbf{r}), \tag{341}$$

$$\Delta E \approx V\mu_0 M^2. \tag{342}$$